CS3621 Introduction to Computing with Geometry
Drill Exercises: Basic Concepts

1. Find the line defined by points (3,5) and (7, 14).

2. What is the distance from the origin to the line with equation $3x + 4y + 6 = 0$?

3. Find the equation of the plane defined by base point $\mathbf{B}$ and normal vector $\mathbf{v}$. Verify your result using $\mathbf{B} = \langle 3, 4, 5 \rangle$ and $\mathbf{n} = \langle 1, 1, 1 \rangle$.

4. What is the inner product and cross product of vectors $\mathbf{u} = \langle 1, 3, 5 \rangle$ and $\mathbf{v} = \langle -2, 0, 4 \rangle$. What is the length of the cross product? What is the angle between these two vectors?

5. Suppose a general second degree curve $Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$ represents a conic curve. Use the concept of *the line at infinity* to prove that the curve is an ellipse, a hyperbola, or a parabola, if and only if $B^2 - AC < 0$, $B^2 - 4AC > 0$, or $B^2 - AC = 0$. This problem looks difficult; but, it should be easy if you understand the concept of the line at infinity covered in class. Try it.

6. Convert the following second degree equation to its equivalent matrix form:

$$3x^2 - 5xy + 5y^2 + x + 6y + 9 = 0$$

7. What is the equation of the following conic curve in matrix form?

$$\begin{bmatrix} -1 & 4 & -3 \\ 4 & 1 & 2 \\ -3 & 2 & 3 \end{bmatrix}$$

8. Convert the following equations to their homogeneous form:

   (a) $x^2 + y^2 + 3z^2 - 2xy + 3xz + yz + 3x - 5y - 6z + 10 = 0$

   (b) $(x^2 + xy^2 - y^2)^2 - 4(x - y^2) - 1 = 0$

   Expanding the given equations could easily cause calculation errors. Try not to expand the second one.

9. Convert the following homogeneous equations to their non-homogeneous form:

   (a) $xy^4 + x^2y^2w + y^4w + 2xyw^3 - 3xw^4 + w^5 = 0$

   (b) $(x^2 + y^2 + zw)^2 + 4w(x^3 + xyw + yw^3) + w^4 = 0$

   Expanding the given equations could easily cause calculation errors. Try not to expand the second one.

10. Find a projective transformation that maps the line determined by two points $(1, 0, 1)$ and $(0, 1, 1)$, in homogeneous coordinate, to the line at infinity.