



Do the following problems using the banker's algorithm:

- Compute what each process might still request and fill this in under the column *Need*.
- Is the system in a safe state? Why or why not?
- Is the system deadlocked? Why or why not?
- If a request from  $P_3$  arrives for  $(2, 1, 0, 0)$ , can the request be granted immediately?

**Always provide your detailed calculation. Or you risk a lower grade.**

**Answer:** We first calculate the *Need* array as follows:

	<i>Allocation</i>				<i>Max</i>				<i>Need</i>				<i>Available</i>			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
$P_0$	2	0	1	1	3	2	1	1	1	2	0	0	6	4	4	2
$P_1$	1	1	0	0	1	2	0	2	0	1	0	2				
$P_2$	1	0	1	0	3	2	1	0	2	2	0	0				
$P_3$	0	1	0	1	2	1	0	1	2	0	0	0				

Because  $P_0$ 's  $Need = [1, 2, 0, 0] \leq Available = [6, 4, 4, 2]$ ,  $P_0$  can run and return its  $Allocation = [2, 0, 1, 1]$  to  $Available$ . The new  $Available = [6, 4, 4, 2] + [2, 0, 1, 1] = [8, 4, 5, 3]$ :

	<i>Allocation</i>				<i>Max</i>				<i>Need</i>				<i>Available</i>			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
$P_0$					3	2	1	1					8	4	5	3
$P_1$	1	1	0	0	1	2	0	2	0	1	0	2				
$P_2$	1	0	1	0	3	2	1	0	2	2	0	0				
$P_3$	0	1	0	1	2	1	0	1	2	0	0	0				

Now we run  $P_1$ . After that  $P_1$  returns its  $Allocation = [1, 1, 0, 0]$  to  $Available$ , and, hence  $Available =$  old  $Available = [8, 4, 5, 3] + P_1$ 's  $Allocation = [1, 1, 0, 0] = [9, 5, 5, 3]$ .

	<i>Allocation</i>				<i>Max</i>				<i>Need</i>				<i>Available</i>			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
$P_0$					3	2	1	1					9	5	5	3
$P_1$					1	2	0	2								
$P_2$	1	0	1	0	3	2	1	0	2	2	0	0				
$P_3$	0	1	0	1	2	1	0	1	2	0	0	0				

Now we can run  $P_2$  followed by  $P_3$ . Consequently, the system is in a safe state. Because the deadlock state is in the unsafe area and because this system is in a safe state, this system is not deadlocked.

Now get back to the original situation:

	<i>Allocation</i>				<i>Max</i>				<i>Need</i>				<i>Available</i>			
	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
$P_0$	2	0	1	1	3	2	1	1	1	2	0	0	6	4	4	2
$P_1$	1	1	0	0	1	2	0	2	0	1	0	2				
$P_2$	1	0	1	0	3	2	1	0	2	2	0	0				
$P_3$	0	1	0	1	2	1	0	1	2	0	0	0				

If a new request from  $P_3$  arrives for  $[2, 1, 0, 0]$ , this is a mistake because  $P_3$ 's  $Request = [2, 1, 0, 0] \not\leq P_3$ 's  $Need = [2, 0, 0, 0]$  and this request cannot be granted. See the Resource-Request algorithm for the details. ■

3. [10 points] Let us revisit the dining philosophers problem again using banker's algorithm. Suppose there are only 3 philosophers  $P_0$ ,  $P_1$  and  $P_2$  and 3 chopsticks  $C_0$ ,  $C_1$  and  $C_2$ . Initially, the system starts with the following:

	Allocation			Max			Need			Available		
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$
$P_0$	0	0	0	1	1	0				1	1	1
$P_1$	0	0	0	0	1	1						
$P_2$	0	0	0	1	0	1						

Again, each philosopher will request his left chopstick followed by his right chopstick. Therefore, philosopher  $P_i$  will do the following:

```

while (true) {
    Request( $C_i$ );
    Request( $C_{(i+1)\%3}$ );
    Eat;
    Release( $C_{(i+1)\%3}$ );
    Release( $C_i$ );
}

```

Note that philosophers may make their requests/releases concurrently; however, you may assume each request and release will be handled in a mutually exclusive way. Use banker's algorithm (*i.e.*, resource-request and safety) to allocate chopsticks so that each philosopher will eat at least twice.

**Always provide your detailed calculation. Or you risk a lower grade.**

**Answer:** This is a problem for you to practice a sequence of requests and releases. Because of a simple deadlock due to all philosophers picking up their left chopsticks at the same time, we shall follow this execution sequence:

- $P_0$  requests left chopstick  $C_0$
- $P_1$  requests left chopstick  $C_1$
- $P_2$  requests left chopstick  $C_2$
- $P_0$  requests right chopstick  $C_1$
- $P_1$  requests right chopstick  $C_2$
- $P_2$  requests right chopstick  $C_0$

If banker's algorithm determines that a particular request cannot be made if doing so will make the system into an unsafe state, we need a queue to store these unsatisfied requests. Then, once a release is made, after updating the *Allocation*, *Need* and *Available*, we will try to satisfy the queued request.

- (a)  $P_0$  requests  $C_0$ : Because  $P_0$ 's *Request* =  $[1, 0, 0] \leq$  *Available* =  $[1, 1, 1]$ , this request can be satisfied. Please find a safe sequence by yourself. After this allocation, the matrix representation becomes:

	Allocation			Max			Need			Available		
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$
$P_0$	1	0	0	1	1	0	0	1	0	0	1	1
$P_1$	0	0	0	0	1	1	0	1	1			
$P_2$	0	0	0	1	0	1	1	0	1			

- (b)  $P_1$  requests  $C_1$ : Because  $P_1$ 's  $Request = [0, 1, 0] \leq Available = [0, 1, 1]$ , this request can be satisfied. Please find a safe sequence by yourself. After this allocation, the matrix representation becomes:

	Allocation			Max			Need			Available		
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$
$P_0$	1	0	0	1	1	0	0	1	0	0	0	1
$P_1$	0	1	0	0	1	1	0	0	1			
$P_2$	0	0	0	1	0	1	1	0	1			

- (c)  $P_2$  requests  $C_2$ :  $P_2$ 's  $Request = [0, 0, 1] \leq Available = [0, 0, 1]$ . Can we satisfy this request? According to the Resource-Request algorithm, pretending this request can be allocated and then updating the tables. In other words,  $P_2$ 's  $Allocation = [0, 0, 0] + Request = [0, 0, 1] = [0, 0, 1]$ ,  $Need = [1, 0, 1] - Request = [0, 0, 1] = [1, 0, 0]$ , and  $Available = [0, 0, 1] - Request = [0, 0, 1] = [0, 0, 0]$ . Therefore, the new matrices are:

	Allocation			Max			Need			Available		
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$
$P_0$	1	0	0	1	1	0	0	1	0	0	0	0
$P_1$	0	1	0	0	1	1	0	0	1			
$P_2$	0	0	1	1	0	1	1	0	0			

It is easily seen that the system is not in a safe state because none of processes can run as their  $Requests$  are all  $\not\leq Available$ . As a result, we have to return to the original tables and queue the request " $P_2$  requests  $C_2$ ".

Now the actual tables are shown below, with the queue of requests:

	Allocation			Max			Need			Available			Queue
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	
$P_0$	1	0	0	1	1	0	0	1	0	0	0	1	$P_2$ request $C_2$
$P_1$	0	1	0	0	1	1	0	0	1				
$P_2$	0	0	0	1	0	1	1	0	1				

- (d)  $P_0$  requests  $C_1$ :  $P_0$ 's  $Request = [0, 1, 0] \not\leq Available = [0, 0, 1]$ . This cannot be satisfied, and has to be queued. The matrix representation is shown below:

	Allocation			Max			Need			Available			Queue
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	
$P_0$	1	0	0	1	1	0	0	1	0	0	0	1	$P_2$ requests $C_2$
$P_1$	0	1	0	0	1	1	0	0	1				$P_0$ requests $C_1$
$P_2$	0	0	0	1	0	1	1	0	1				

- (e)  $P_1$  requests  $C_2$ :  $P_1$ 's  $Request = [0, 0, 1] \leq Available = [0, 0, 1]$ . Let us do a pre-allocation check with banker's algorithm.  $P_1$ 's new  $Allocation = [0, 1, 1]$ ,  $P_1$ 's new  $Need = [0, 0, 0]$  and  $Available = [0, 0, 0]$ .

	Allocation			Max			Need			Available		
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$
$P_0$	1	0	0	1	1	0	0	1	0	0	0	0
$P_1$	0	1	1	0	1	1	0	0	0			
$P_2$	0	0	0	1	0	1	1	0	1			

We can easily find a safe sequence. For example, because  $P_1$ 's  $Need = [0, 0, 0] \leq Available = [0, 0, 0]$ , it can run and return  $P_1$ 's  $Allocation = [0, 1, 1]$  making  $Available = [0, 0, 0] + [0, 1, 1]$ . This is equivalent to say that giving  $P_1$  its right chopstick allowing  $P_1$  to eat and releasing both chopsticks. After  $Available = [0, 1, 1]$ , then  $P_0$ 's  $Request = [0, 1, 0]$  can be satisfied. Finally,  $P_2$  can run. Consequently, we have found a safe sequence  $\langle P_1, P_0, P_2 \rangle$ . Therefore, this request can be satisfied and the matrix representation becomes:

	Allocation			Max			Need			Available			Queue
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	
$P_0$	1	0	0	1	1	0	0	1	0	0	0	0	$P_2$ requests $C_2$
$P_1$	0	1	1	0	1	1	0	0	0				$P_0$ requests $C_1$
$P_2$	0	0	0	1	0	1	1	0	1				

- (f)  $P_1$  releases  $C_1$  and  $C_2$ :  $P_1$  returns  $C_1$  and  $C_2$ , making its  $Allocation = [0, 0, 0]$ ,  $Request = [0, 0, 0]$  and  $Available = [0, 1, 1]$ . Now the new matrix representation is:

	Allocation			Max			Need			Available			Queue
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	
$P_0$	1	0	0	1	1	0	0	1	0	0	1	1	$P_2$ requests $C_2$
$P_1$	0	0	0	0	1	1	0	0	0				$P_0$ requests $C_1$
$P_2$	0	0	0	1	0	1	1	0	1				

Now let us handle *Queue*. The first is  $P_2$ 's  $Request = [0, 0, 1]$ . Can we satisfy this request? Pretending we can do that. Then,  $P_2$ 's  $Allocation = [0, 0, 1]$  (i.e., getting  $C_2$  – left chopstick),  $P_2$ 's  $Need = [1, 0, 1] - [0, 0, 1] = [1, 0, 0]$ , and  $Available = [0, 1, 0]$ . This is equivalent to say that philosophers  $P_0$  and  $P_2$  have their left chopsticks  $C_0$  and  $C_2$ , respectively.

	Allocation			Max			Need			Available		
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$
$P_0$	1	0	0	1	1	0	0	1	0	0	1	0
$P_1$	0	0	0	0	1	1	0	0	0			
$P_2$	0	0	1	1	0	1	1	0	0			

We can easily find a safe sequence  $\langle P_1, P_0, P_2 \rangle$ . This is equivalent to say that giving  $C_1$  to philosopher  $P_0$ , and after  $P_0$  finishes eating, philosopher  $P_2$  can take  $P_0$ 's left chopstick  $C_0$ , which is  $P_2$ 's right chopstick. Finally,  $P_2$  has both chopsticks and eat!

After this allocation, we have the following matrix representation:

	Allocation			Max			Need			Available			Queue
	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	$C_0$	$C_1$	$C_2$	
$P_0$	1	0	0	1	1	0	0	1	0	0	1	0	$P_0$ requests $C_1$
$P_1$	0	0	0	0	1	1	0	0	0				
$P_2$	0	0	0	1	0	1	1	0	0				

Then, we may allow  $P_0$  to eat, followed by  $P_2$ , etc. This finishes one cycle of eating. This next cycle is similar and is not repeated. ■

4. [10 points] Consider the following snapshot of a system in which five resources  $A, B, C, D$  and  $E$  are available. The system contains a total of 2 instances of  $A$ , 1 of resource  $B$ , 1 of resource  $C$ , 2 resource  $D$  and 1 of resource  $E$ .

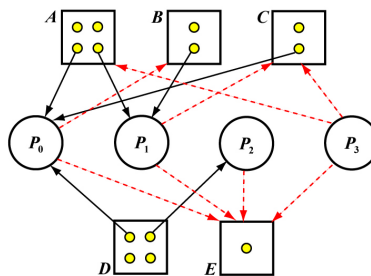
	Allocation					Request					Available				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
$P_0$	1	0	1	1	0	0	1	0	0	1	2	1	1	2	1
$P_1$	1	1	0	0	0	0	0	1	0	1					
$P_2$	0	0	0	1	0	0	0	0	0	1					
$P_3$	0	0	0	0	0	1	0	1	0	1					

Do the following problems:

- Convert this matrix representation to a resource allocation graph.
- Use the deadlock detection algorithm to determine whether the system contains a deadlock. Which processes are involved in the deadlock?
- While you are use the deadlock detection algorithm, add and remove directed edges of the resource allocation graph.

**Always provide your detailed calculation. Or you risk a lower grade.**

**Answer:** The corresponding resource allocation graph is shown below:

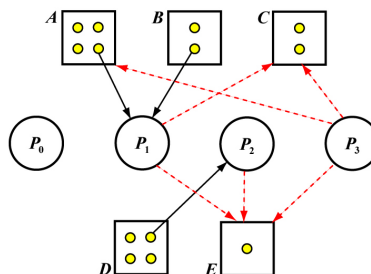


Note that red dashed-lined arrows are used to indicate unsatisfied “requests”.

Currently, because  $P_0$ 's  $Request = [0, 1, 0, 0, 1] \leq Available = [2, 1, 1, 2, 1]$ , we run  $P_0$  and reclaim its  $Allocation$ , and the new  $Allocation = Old Allocation + Available = [1, 0, 1, 1, 0] + [2, 1, 1, 2, 1] = [3, 1, 2, 3, 1]$ . The new matrix representation becomes:

	Allocation					Request					Available				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
$P_0$											3	1	2	3	1
$P_1$	1	1	0	0	0	0	0	1	0	1					
$P_2$	0	0	0	1	0	0	0	0	0	1					
$P_3$	0	0	0	0	0	1	0	1	0	1					

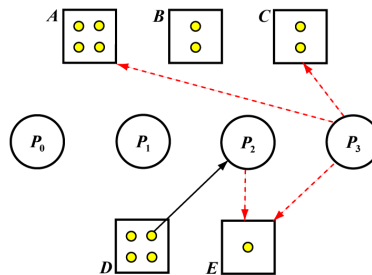
For the resource allocation graph, because all resources allocated to  $P_0$  are returned and  $P_0$  has no new request (so far), the new graph is obtained by removing all request edges and allocation edges as shown below:



Now, we have  $P_1$ 's Request =  $[0, 0, 1, 0, 1] \leq Available = [3, 1, 2, 3, 1]$ . We can run  $P_1$ , and reclaim its Allocation =  $[1, 1, 0, 0, 0]$ . The new Available =  $[3, 1, 2, 3, 1] + P_1$ 's Allocation =  $[1, 1, 0, 0, 0] = [4, 2, 2, 3, 1]$ . As a result, the new matrix representation is:

	Allocation					Request					Available				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
$P_0$											4	2	2	3	1
$P_1$															
$P_2$	0	0	0	1	0	0	0	0	0	1					
$P_3$	0	0	0	0	0	1	0	1	0	1					

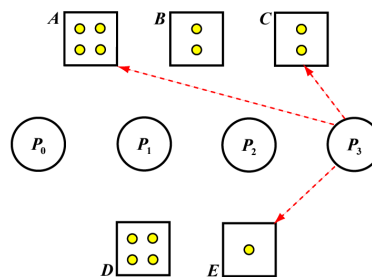
By removing all edges of  $P_1$  yields a new resource allocation graph:



Next, we run  $P_2$  and the new matrix representation and resource allocation graph are:

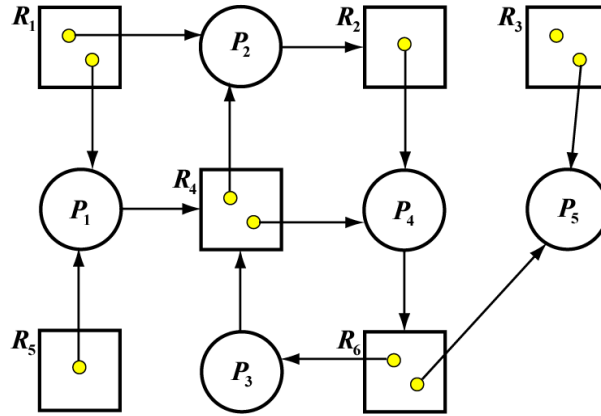
	Allocation					Request					Available				
	A	B	C	D	E	A	B	C	D	E	A	B	C	D	E
$P_0$											4	2	2	4	1
$P_1$															
$P_2$															
$P_3$	0	0	0	0	0	1	0	1	0	1					

Removing all edges from and to  $P_1$  yields a new resource allocation graph:



Finally,  $P_3$  can run and return all of its resources. Because all involved processes are finished, this system does not have a deadlock. ■

5. [10 points] Consider the following resource allocation graph.



Do the following problems:

- (a) Convert it to the matrix representation (i.e., Allocation, Request and Available).
- (b) Do a step-by-step execution of the deadlock detection algorithm. For each step, add and remove the directed edges, and redraw the resource allocation graph.
- (c) Is there a deadlock? If there is a deadlock, which processes are involved?

**Always provide your detailed calculation. Or you risk a lower grade.**

**Answer:** Note that the given resource allocation graph has cycles:  $P_2 \rightarrow R_2 \rightarrow P_4 \rightarrow R_6 \rightarrow P_3 \rightarrow R_4 \rightarrow P_2$  and  $P_4 \rightarrow R_6 \rightarrow P_3 \rightarrow R_4 \rightarrow P_4$ . The following is the corresponding matrix representation.

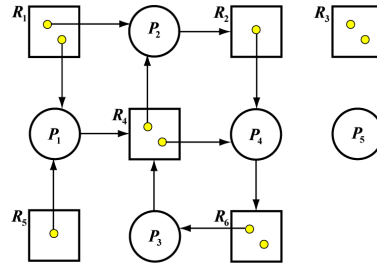
	Allocation						Request						Available						
	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	
$P_1$	1	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
$P_2$	1	0	0	1	0	0	0	1	0	0	0	0							
$P_3$	0	0	0	0	0	1	0	0	0	1	0	0							
$P_4$	0	1	0	1	0	0	0	0	0	0	0	1							
$P_5$	0	0	1	0	0	1	0	0	0	0	0	0							

We can run the system based on the deadlock detection algorithm. Note that other than  $R_3$  there is no instance available in  $R_1, R_2, R_4, R_5$  and  $R_6$ . The only runnable process is  $P_5$ . Because  $P_5$ 's  $Request = [0, 0, 0, 0, 0, 0] \leq Available = [0, 0, 1, 0, 0, 0]$ , we can run  $P_5$  and reclaim its allocated resource. The new  $Available$  becomes  $Available = old\ Available + P_5's\ Allocation = [0, 0, 1, 0, 0, 0] + [0, 0, 1, 0, 0, 1] = [0, 0, 2, 0, 0, 1]$ .

	Allocation						Request						Available						
	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	
$P_1$	1	0	0	0	1	0	0	0	0	1	0	0	0	0	2	0	0	0	1
$P_2$	1	0	0	1	0	0	0	1	0	0	0	0							
$P_3$	0	0	0	0	0	1	0	0	0	1	0	0							
$P_4$	0	1	0	1	0	0	0	0	0	0	0	1							
$P_5$																			

Removing all edges related to  $P_5$  gives the following resource allocation graph:

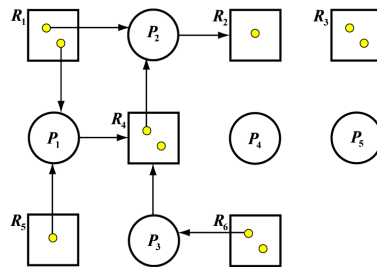




Because the current  $Available = [0, 0, 2, 0, 0, 1] \geq P_4's Request = [0, 0, 0, 0, 0, 1]$ , we can run  $P_4$  and reclaim  $P_4$ 's allocation  $[0, 1, 0, 1, 0, 0]$ . The new matrix representation is:

	Allocation						Request						Available					
	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$
$P_1$	1	0	0	0	1	0	0	0	0	1	0	0	0	1	2	1	0	1
$P_2$	1	0	0	1	0	0	0	1	0	0	0	0						
$P_3$	0	0	0	0	0	1	0	0	0	1	0	0						
$P_4$																		
$P_5$																		

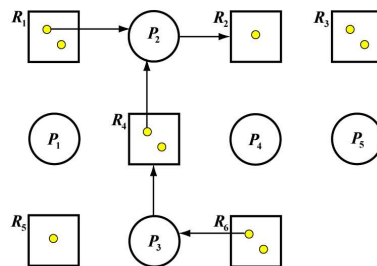
The new version of resource allocation graph is:



Now we can run  $P_1$  because  $P_1's Request = [0, 0, 0, 1, 0, 0] \leq Available = [0, 1, 2, 1, 0, 1]$ . After  $P_1$  finishes its work, the new  $Available = old Available + P_1's Allocation = [0, 1, 2, 1, 0, 1] + [1, 0, 0, 0, 1, 0] = [1, 1, 2, 1, 1, 1]$ . Thus, we have the following new matrix representation:

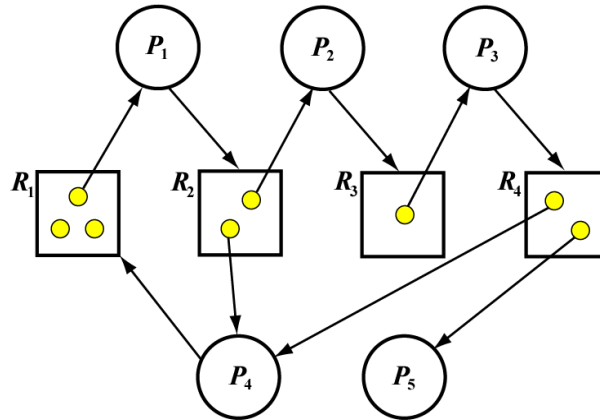
	Allocation						Request						Available					
	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$R_6$
$P_1$													1	1	2	1	1	1
$P_2$	1	0	0	1	0	0	0	1	0	0	0	0						
$P_3$	0	0	0	0	0	1	0	0	0	1	0	0						
$P_4$																		
$P_5$																		

The resource allocation graph becomes:



Next, we run  $P_2$  followed by  $P_3$ . In this way, all processes can run and obtain and release the needed resources. Consequently, there is no deadlock in the system. Thus, even though there are cycles in the given resource allocation graph, there is no deadlock! ■

6. [10 points] Consider the following resource allocation graph.



Do the following problems:

- (a) Convert it to the matrix representation (i.e., Allocation, request and Available).
- (b) Do a step-by-step execution of the deadlock detection algorithm. For each step, add and remove the directed edges, and redraw the resource allocation graph.
- (c) Is there a deadlock? If there is a deadlock, which processes are involved?

**Always provide your detailed calculation. Or you risk a lower grade.**

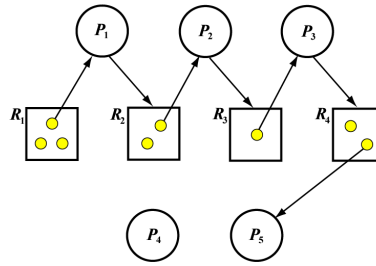
**Answer:** The matrix representation of the given resource allocation graph is shown below:

	Allocation				Request				Available			
	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$
$P_1$	1	0	0	0	0	1	0	0	2	0	0	0
$P_2$	0	1	0	0	0	0	1	0				
$P_3$	0	0	1	0	0	0	0	1				
$P_4$	0	1	0	1	1	0	0	0				
$P_5$	0	0	0	1	0	0	0	0				

Because  $P_4$ 's Request =  $[1, 0, 0, 0] \leq$  Available =  $[2, 0, 0, 0]$ ,  $P_4$  runs and returns is Allocation =  $[0, 1, 0, 1]$  making the new Available =  $[2, 0, 0, 0] + [0, 1, 0, 1] = [2, 1, 0, 1]$ . The matrix representation becomes:

	Allocation				Request				Available			
	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$
$P_1$	1	0	0	0	0	1	0	0	2	1	0	1
$P_2$	0	1	0	0	0	0	1	0				
$P_3$	0	0	1	0	0	0	0	1				
$P_4$												
$P_5$	0	0	0	1	0	0	0	0				

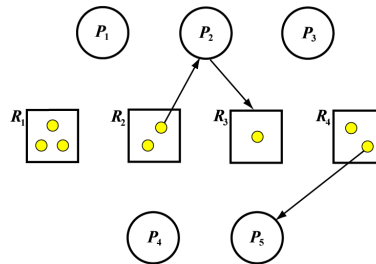
This is the corresponding resource allocation graph:



Then, we can run  $P_1$  because  $P_1$ 's  $Request = [0, 1, 0, 0] \leq Available = [2, 1, 0, 1]$ . After reclaiming  $P_1$ 's  $Allocation = [1, 0, 0, 0]$ , the new  $Available$  is old  $Available = [2, 1, 0, 1] + P_1$ 's  $Allocation = [1, 0, 0, 0] = [3, 1, 0, 1]$ . The new matrix representation is:

	Allocation				Request				Available			
	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$
$P_1$									3	1	0	1
$P_2$	0	1	0	0	0	0	1	0				
$P_3$	0	0	1	0	0	0	0	1				
$P_4$												
$P_5$	0	0	0	1	0	0	0	0				

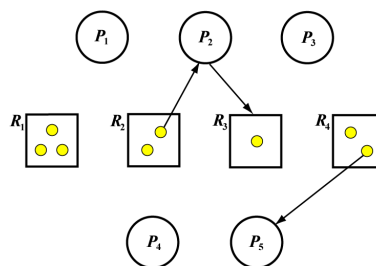
Here is the corresponding resource allocation graph:



The next process is  $P_3$  because  $P_3$ 's  $Request = [0, 0, 0, 1] \leq Available = [3, 1, 0, 1]$ . After  $P_3$  finishes its work, its  $Allocation = [0, 0, 1, 0]$  is returned to  $Available = [3, 1, 0, 1] + [0, 0, 1, 0] = [3, 1, 1, 1]$ :

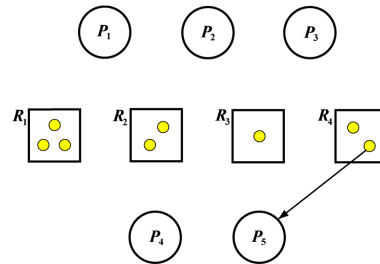
	Allocation				Request				Available			
	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$
$P_1$									3	1	1	1
$P_2$	0	1	0	0	0	0	1	0				
$P_3$												
$P_4$												
$P_5$	0	0	0	1	0	0	0	0				

The resource allocation graph is shown below:



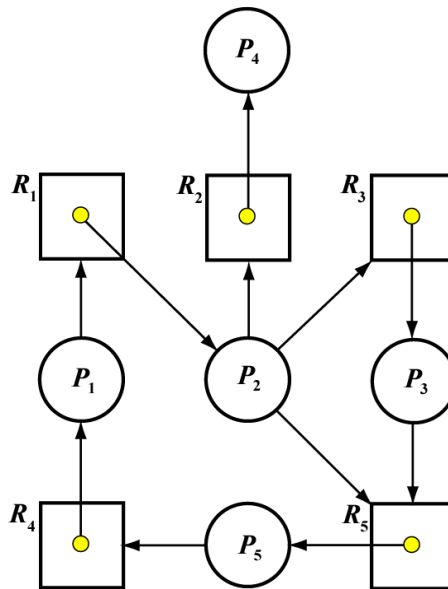
Now we can run  $P_2$  and the yields (*i.e.*, matrix representation and resource allocation graph) are:

	Allocation				Request				Available			
	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$	$R_1$	$R_2$	$R_3$	$R_4$
$P_1$									3	2	1	1
$P_2$												
$P_3$												
$P_4$												
$P_5$	0	0	0	1	0	0	0	0				



Finally, we can run  $P_5$  and all processes are done! Note that there are cycles  $P_1 \rightarrow R_2 \rightarrow P_4 \rightarrow R_1 \rightarrow P_1$  and  $P_1 \rightarrow R_2 \rightarrow P_2 \rightarrow R_3 \rightarrow P_3 \rightarrow R_4 \rightarrow P_4 \rightarrow R_1 \rightarrow P_1$ . However, there is no deadlock. ■

7. [10 points] Consider the following resource allocation graph.



Do the following problems:

- Convert it to the matrix representation (*i.e.*, Allocation, Request and Available).
- Do a step-by-step execution of the deadlock detection algorithm. For each step, add and remove the directed edges, and redraw the resource allocation graph.
- Is there a deadlock? If there is a deadlock, which processes are involved?

**Always provide your detailed calculation. Or you risk a lower grade.**

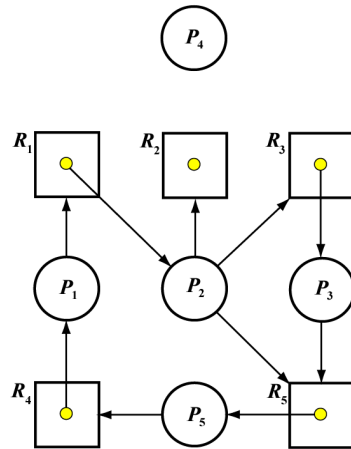
**Answer:** The matrix representation of the given resource allocation graph is shown below:

	Allocation					Request					Available				
	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>
P <sub>1</sub>	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
P <sub>2</sub>	1	0	0	0	0	0	1	1	0	1					
P <sub>3</sub>	0	0	1	0	0	0	0	0	0	1					
P <sub>4</sub>	0	1	0	0	0	0	0	0	0	0					
P <sub>5</sub>	0	0	0	0	1	0	0	0	1	0					

P<sub>4</sub> can run because its Request = [0, 0, 0, 0, 0] ≤ Available = [0, 0, 0, 0, 0]. The new Available is calculated as old Available = [0, 0, 0, 0, 0] + P<sub>4</sub>'s Allocation = [0, 1, 0, 0, 0] = [0, 1, 0, 0, 0]. The new matrix representation is

	Allocation					Request					Available				
	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>
P <sub>1</sub>	0	0	0	1	0	1	0	0	0	0	0	1	0	0	0
P <sub>2</sub>	1	0	0	0	0	0	1	1	0	1					
P <sub>3</sub>	0	0	1	0	0	0	0	0	0	1					
P <sub>4</sub>															
P <sub>5</sub>	0	0	0	0	1	0	0	0	1	0					

The corresponding resource allocation graph is:



None of the remaining processes (i.e., P<sub>1</sub>, P<sub>2</sub>, P<sub>3</sub> and P<sub>5</sub>) can run. As a result, we have a deadlock in this system. Note that we have cycles: P<sub>1</sub> → R<sub>1</sub> → P<sub>2</sub> → R<sub>5</sub> → P<sub>5</sub> → R<sub>4</sub> → P<sub>1</sub> and P<sub>1</sub> → R<sub>1</sub> → P<sub>2</sub> → R<sub>3</sub> → P<sub>3</sub> → R<sub>5</sub> → P<sub>5</sub> → R<sub>4</sub> → P<sub>1</sub>. Thus, without actually working on a resource allocation graph to reduce it to its final form, having a cycle does not always mean there is a deadlock. ■