Random Trip Mobility Models

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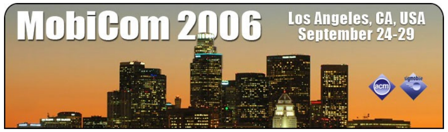

Tutorial ACM Mobicom 2006

Resources

- Random trip model web page:

  http://ica1www.epfl.ch/RandomTrip

  - Links to slides, papers, perfect simulation software

- This tutorial is mainly based on:

  The Random Trip Model: Stability, Stationary Regime, and Perfect Simulation, ACM/IEEE Trans. on Networking, to appear Dec 06

  - Extended journal version of IEEE Infocom 2005 paper
  - Technical report with proofs: MSR-TR-2006-26

Abstract

Mobility models play an important role for wireless and mobile systems as they are used widely for both mathematical and simulation-based evaluations. Even though some of mobility models are rather simple, such as for example well known random waypoint model, they often cause some subtle problems. For example, the annoying initial transience of node mobility state, and the decrease of node numerical speed to zero during a simulation run. Some of these issues were addressed in the literature on a case by case basis, often involving long and complicated computations, which blur understanding the roots of the experienced problems and ways to fix them. It is critical to perform simulations that are free of biases such as initial transience and avoid abnormal cases such as the speed decay to zero in order to produce fair comparative performance of protocols in mobile environments.

In the tutorial, we present random trip models, a broad class of random mobility models and review a large number of random trip model examples, such as for example, random waypoint on convex or non convex areas, restricted random waypoint, inter-city, space graph, boundary reflection and wrap-around models. Our first goal is to explain the trip conditions that define random trip mobility models and guarantee the model stability. The stability is in the sense of existence of time stationary mobility state and convergence of the node mobility state to a unique time-stationary state, from any initial node mobility state. Knowing such conditions is important in order to enable verification of stability of existing and new mobility models and by doing so, avoiding undesirable phenomena such as the aforementioned speed decay to zero. The stability conditions originate from the theory of continuous-time Markov processes on general state spaces; this framework is rather delicate but we explain the stability conditions in an easy way that suffices to apply them.

cont'd

Abstract (2)

We further present perfect simulation algorithm that initialises node mobility state in a way that the state remains time-stationary throughout a simulation run - hence, perfect simulation. This is rather useful as it entirely alleviates the annoying initial transience of node mobility state. The algorithm does not necessitate knowing the mean trip duration for all trips, but it suffices to know a bound on the mean trip duration in cases when the mean trip duration is difficult to compute. This is rather relevant in practise as computing the mean trip duration typically involves computing geometric constants that are often hard to compute, while computing close bounds on the mean trip duration is often easy. We describe how to use the implementation of perfect simulation algorithm to use with ns-2 that is freely available for download. This tool has been used by others in performance evaluations of some recent wireless and mobile systems.

We lastly discuss how random trip mobility model accommodates various mobility properties (some of which may be invariants of real-world mobility) such as, for example, recent empirical evidence that the distribution of human inter-contact times are heavy-tailed, long-range dependent models and their implications on simulation averaging, and parameter settings of node mobility to achieve a target time-stationary distribution of node location. We also point to some data resources to use with the model towards realistic mobility simulations.

cont'd

Abstract (3)

Audience

Researchers, systems people, and students who want to learn or better understand the state-of-the art mobility models, their stability, stationary regime, convergence properties, and perfect simulation. The attendees will learn the framework that defines random trip mobility models, which would enable them defining new mobility models with guaranteed stability and convergence properties, so as to avoid pitfalls such as for example experienced with random waypoint model. They will also learn how to run perfect simulations of random trip mobility models, which will be supported by demonstration of the software tool designed to use with ns2 simulator. No special background is assumed, but some basic familiarity with applied probability.

Why this tutorial ?

- Mobility models are used for performance evaluation of mobile systems by many
  - Simulations
  - Maths

- Experience with simulations is intriguing
  - Speed decay: average speed decays with simulation time
  - Initial transience: different initial and long-run distributions

- Origins of issues
  - Model definition (stability)
  - Simulation technique (initial sample)

Why this tutorial ? (2)

- Critical to adopt best simulation practices
  - Make sure model is stable
    (avoid speed decay and similar abnormal cases)
  - Run stationary simulations, if possible
    (avoid annoying initial transience)

Outline

- Simulation Issues with mobility models
- Random trip basic constructs
- A technical condition: Positive Harris recurrence
- Stability of random trip model
- Time-stationary distributions
- Perfect simulation
- FAQ

Outline

- FAQ
  - Does model accommodate power-law inter-contact times ?
  - Does model accommodate heavy-tailed trip durations ?
  - Can model produce a given time-stationary distribution of node position ?
  - What are mobility data resources ?

Outline

Simulation Issues with mobility models

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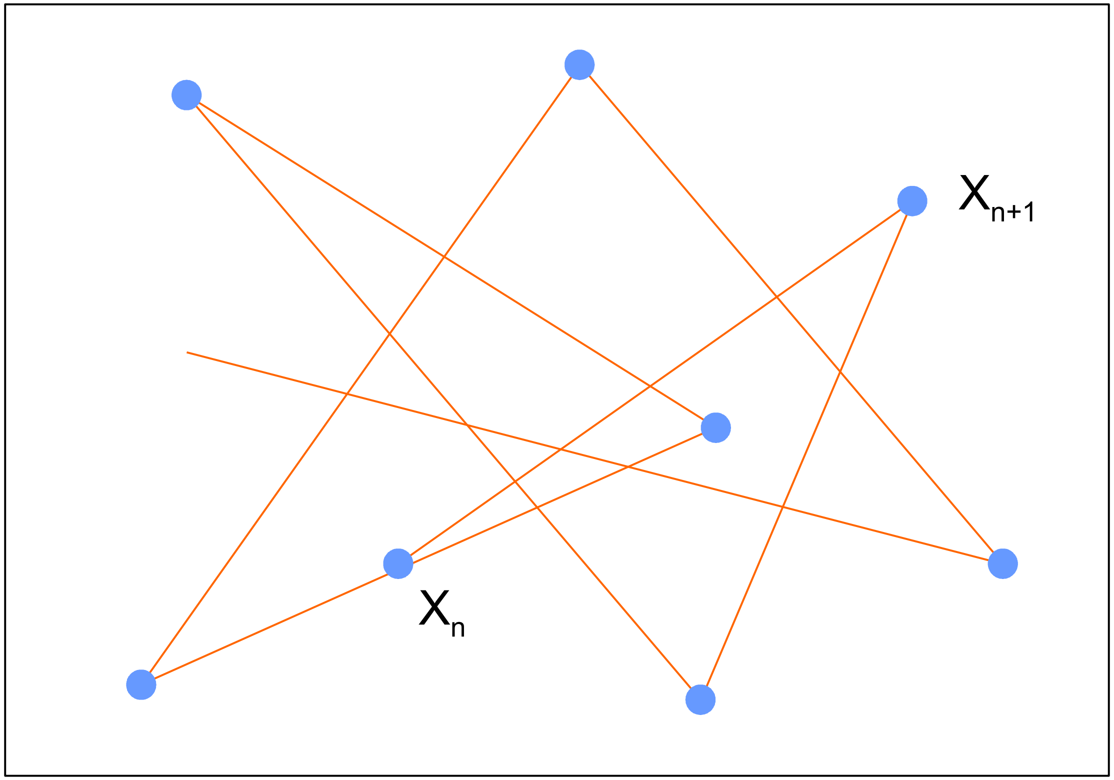
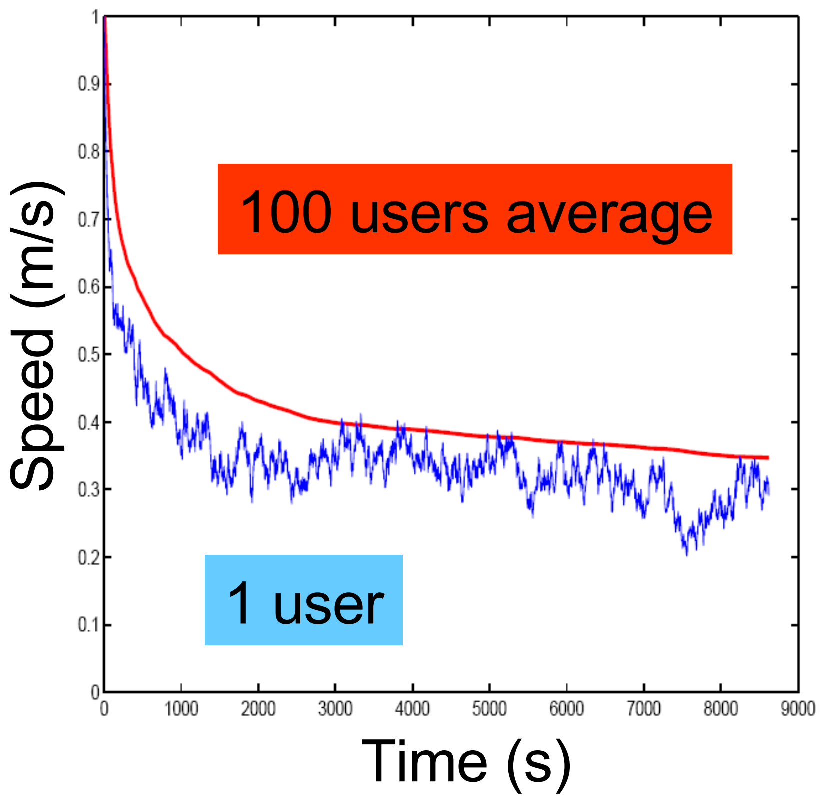
Simplest example: random waypoint (Johnson and Maltz`96)

- Node:
  - Picks next waypoint X_{n+1} uniformly in area
  - Picks speed V_n uniformly in $[v_{min},v_{max}]$
  - Moves to X_{n+1} with speed V_n



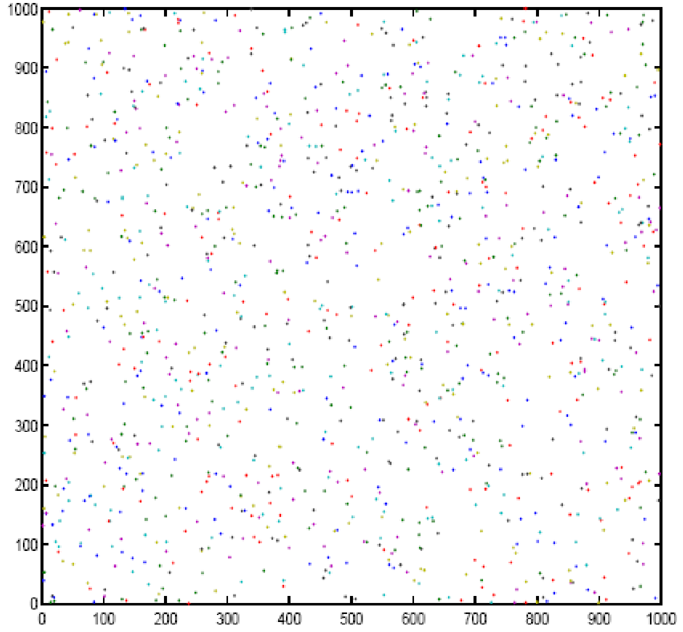
Already the simple model exhibits issues

- Distributions of node speed, position, distances, etc change with time
  - Node speed:

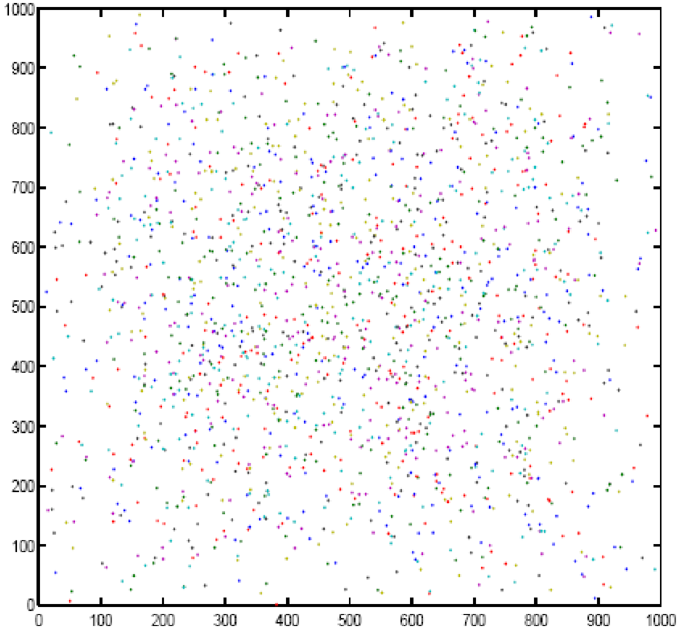


Already the simple model exhibits issues (2)

- Distributions of node speed, position, distances, etc change with time
  - Distribution of node position:



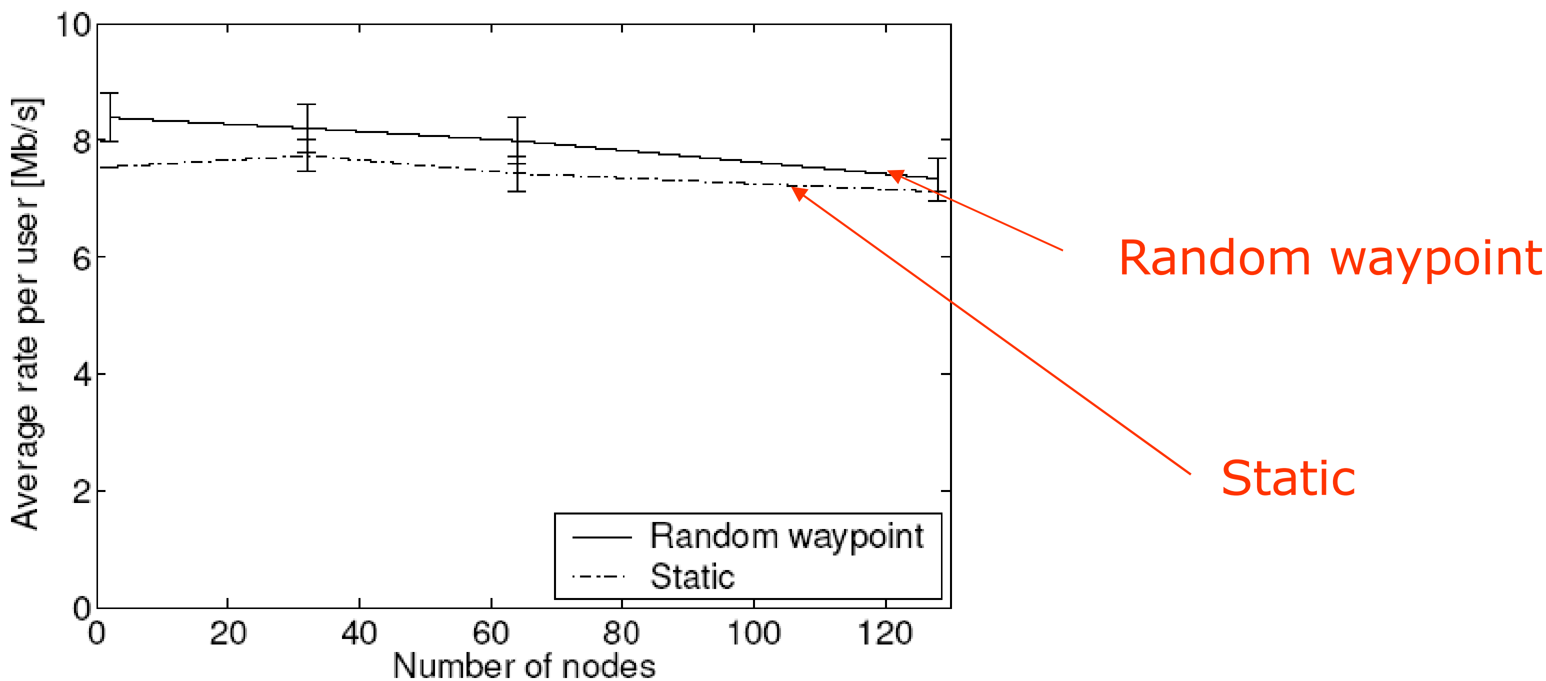
Time = 0 sec



Time = 2000 sec

Why does it matter ?

- A (true) example: Compare impact of mobility on a protocol:
  - Experimenter places nodes uniformly for static case, according to random waypoint for mobile case
  - Finds that static is better
- **Q.** Find the bug !



- **A.** In the mobile case, the nodes are more often towards the center, distance between nodes is shorter, performance is better
- The comparison is flawed. Should use for static case the same distribution of node location as random waypoint. *Is there such a distribution to compare against ?*

Issues with Mobility Models

- Is there a stable distribution of the simulation state (*time-stationary distribution*), reached if we run the simulation long enough ?

- If so:

  - How long is long enough ?

  - If it is too long, is there a way to get to the stable distribution without running long simulations *(perfect simulation)* ?

This tutorial: random trip model

- A broad model of independent node movements
  - Including RWP, realistic city maps, etc
- Defined by a set of conditions on trip selection
- Conditions ensure issues mentioned above are under control
  - Model stability (defined later)
  - Model permits perfect simulation
    - Algorithm in this slide deck
    - Perfect simulation = distribution of node mobility is time-stationary throughout a simulation

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- Simulation Issues with mobility models

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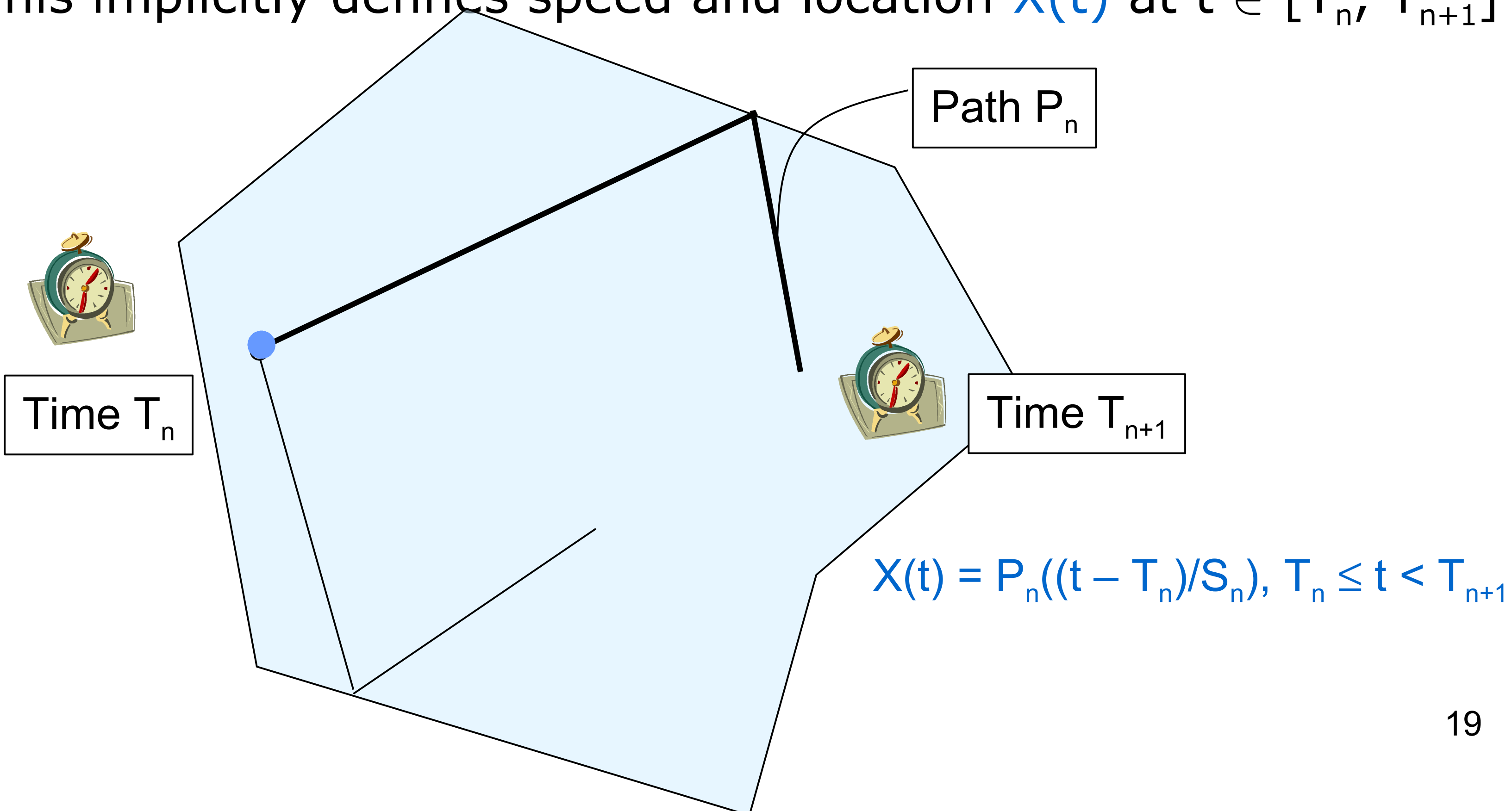
Random trip basic constructs » Outline «

- Initially: a mobile picks a trip, i.e. a combination of 3 elements
  - A path in a catalogue of paths
  - A duration
  - A phase

- A end of trip, mobile picks a new trip
  - Using a trip selection rule
  - Information required to sample next trip is entirely contained in path and phase of previous trip the trip that just finished (*Markov* property)

Illustration of basic constructs

- At end of *(n-1)st* trip, at time T_n, mobile picks
  - Path P_n
  - Duration $S_n = T_{n+1} - T_n$
  - (also a phase – see later)
  - This implicitly defines speed and location X(t) at $t \in [T_n, T_{n+1}]$



Random waypoint is a random trip model

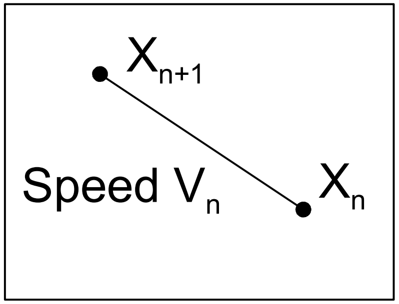
- (Assume in this slide model without pause)
- At end of trip *n-1*, mobile is at location X_n
  - Sample location X_{n+1} uniformly in area

    Path P_n is shortest path from X_n to X_{n+1}

    $P_n(u) = (1 - u) X_n + u X_{n+1}$ for $u \in [0,1]$

  - Sample numerical speed $V_n \geq 0$ from a given speed distribution

    This defines duration:
    $S_n = ||X_{n+1} - X_n|| / V_n$

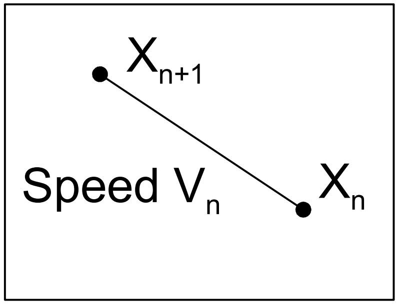


- (Markov property): Information required to sample next trip (location X_n) is entirely contained in path and phase of previous trip

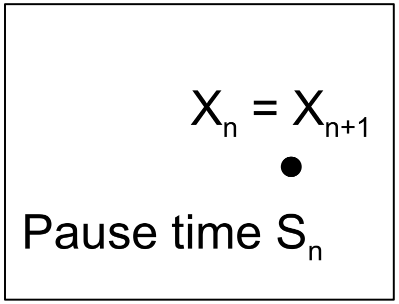
Random waypoint *with pauses* is a random trip model

- Phase I_n is either move or pause
- At end of trip *n-1*:

  If phase I_{n-1} was pause then
  - I_n = move (next trip is a move)
  - Sample X_{n+1} and V_n as on previous slide



  Else
  - I_n = pause (next trip is a move)
  - Path: $P_n(u) = X_n$ for $u \in [0,1]$
  - Pick S_n from a given pause time distribution



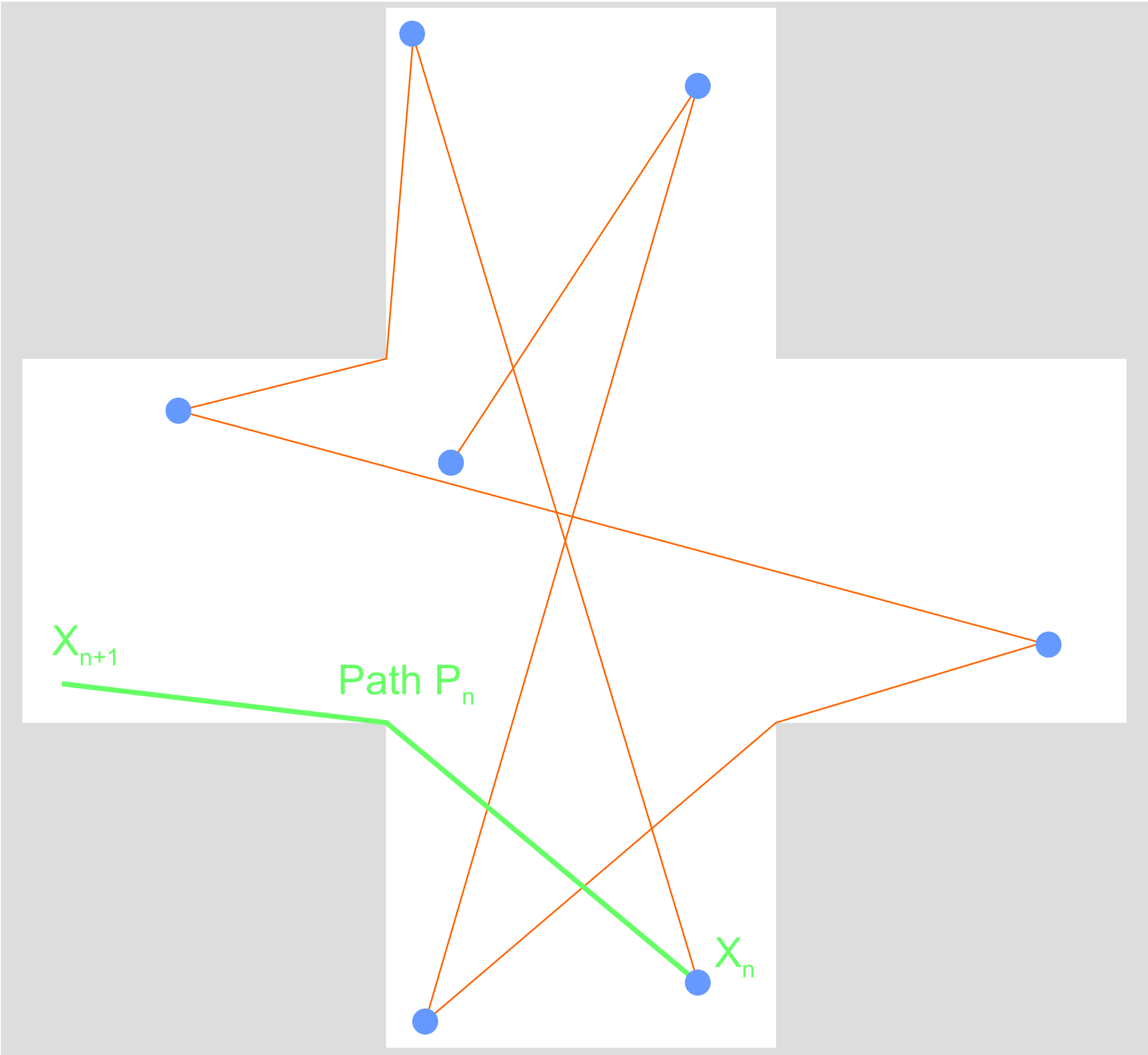
- (Markov property): Information required to sample next trip (phase I_n, location X_n) is entirely contained in path and phase of previous trip

Catalogue of examples

- Random waypoint on general connected domains
  - Swiss Flag
  - City-section
- Restricted random waypoint
  - Inter-city
  - Space-graph
- Random walk on torus
- Billiards
- Stochastic billiards

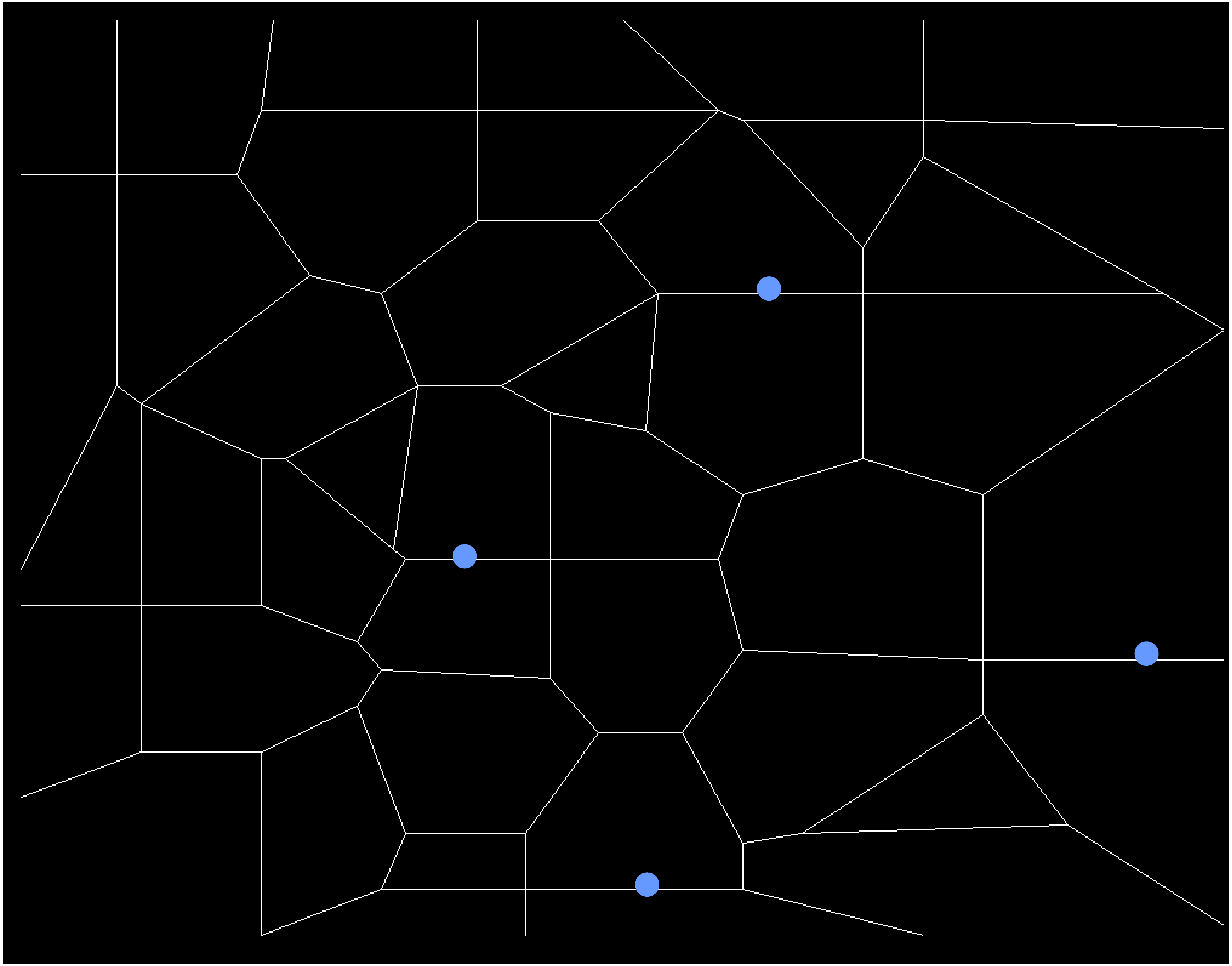
Random waypoint on general connected domain

- Swiss Flag [LV05]
- Non convex domain



Random waypoint on general connected domain (2)

- City-section, Camp et al [CBD02]



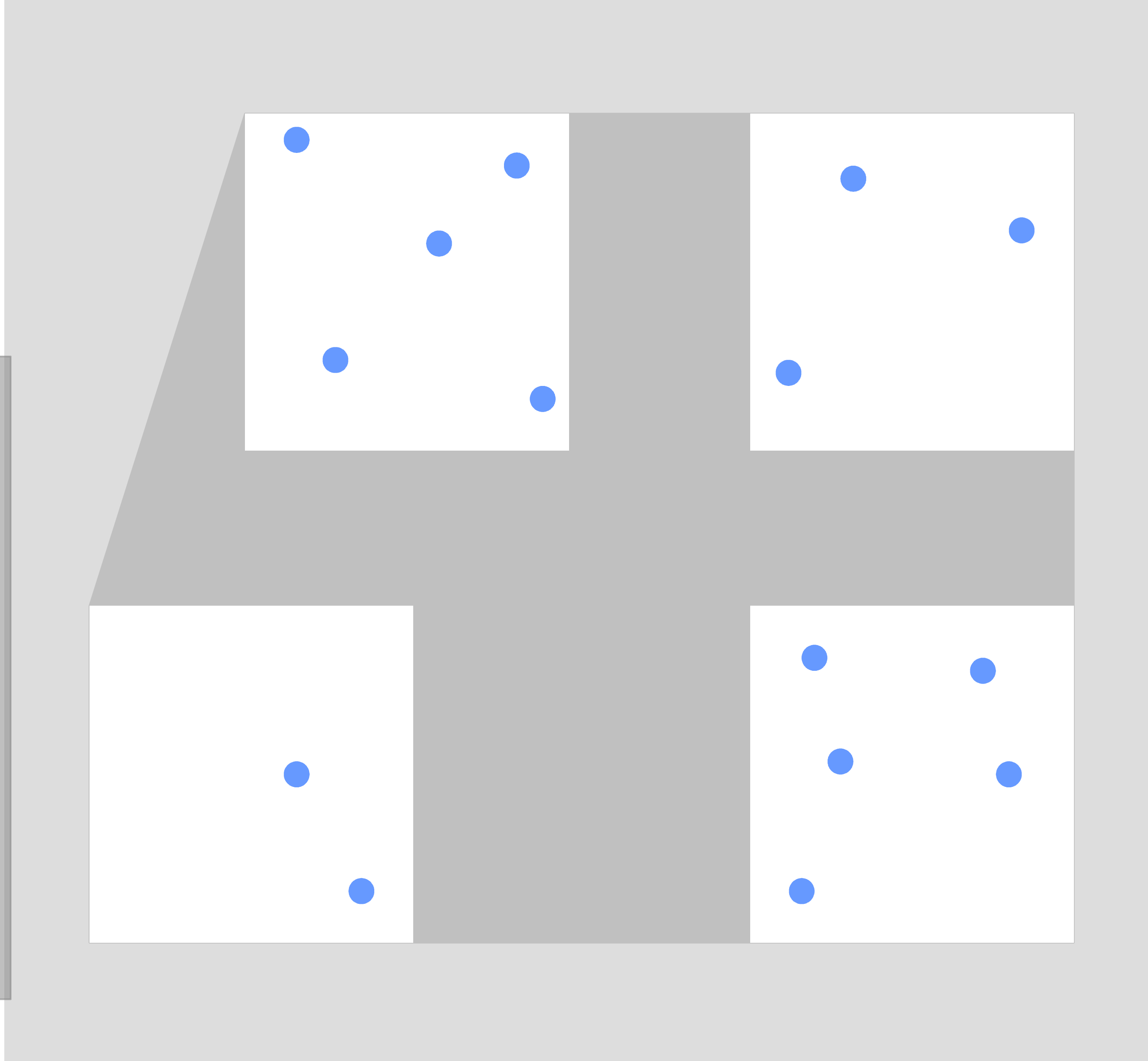
Restricted random waypoint

- Inter-city, Blazevic et al [BGL04]
- Stay in one subdomain for some time then move to other

Here phase is
(I_n, L_n, L_{n+1}, R_n)

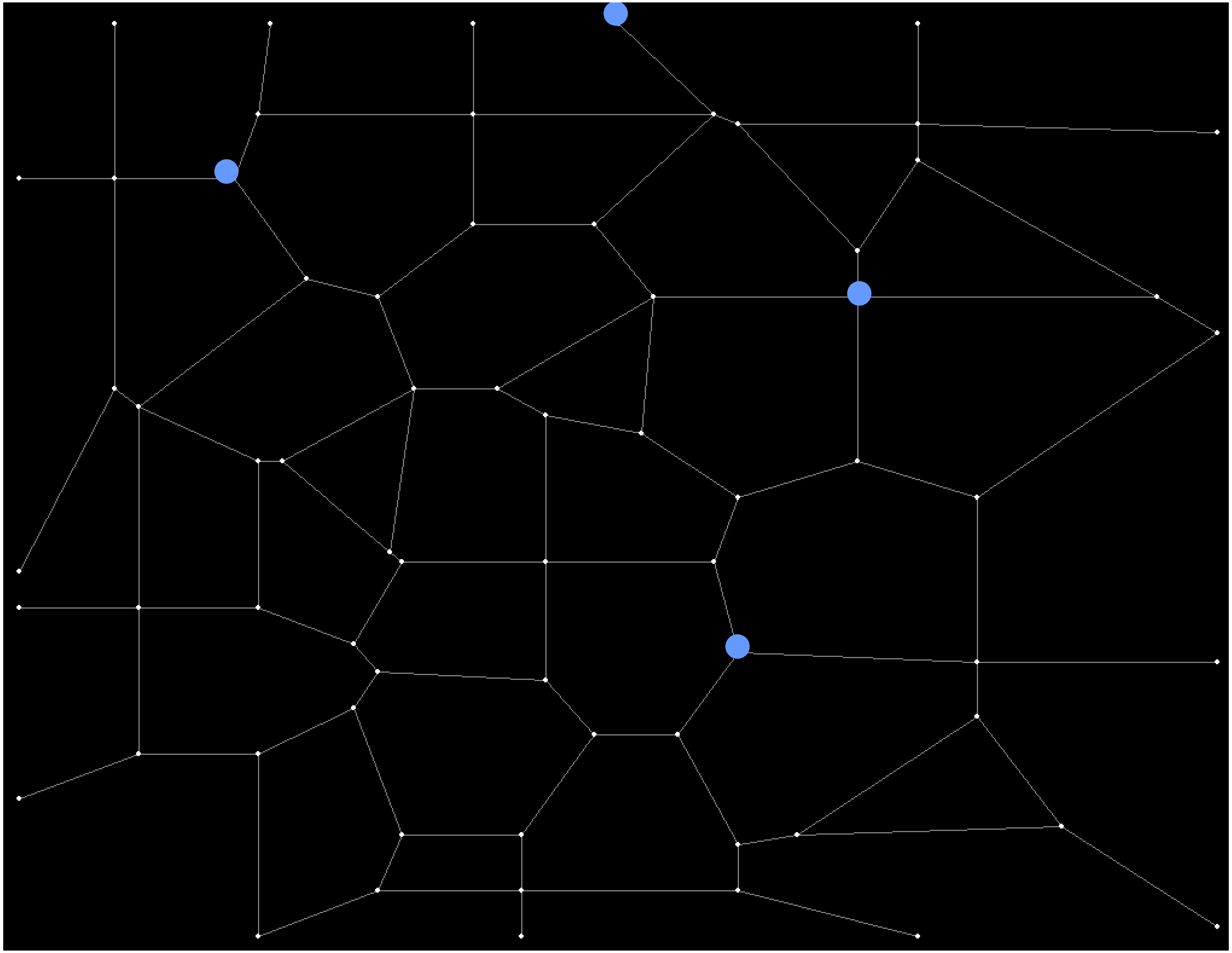
where

I_n = pause or move
L_n = current sub-domain
L_{n+1} = next subdomain
R_n = number of trips in this visit to the current domain



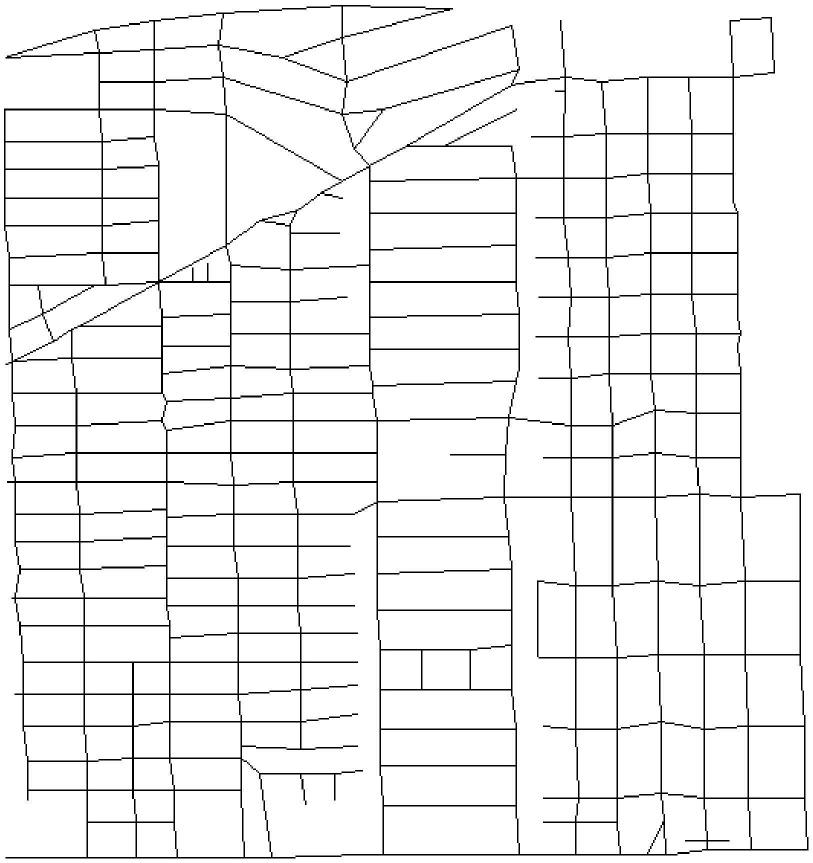
Restricted random waypoint (2)

- Space-graph, Jardosh et al, ACM Mobicom 03 [JBAS03]



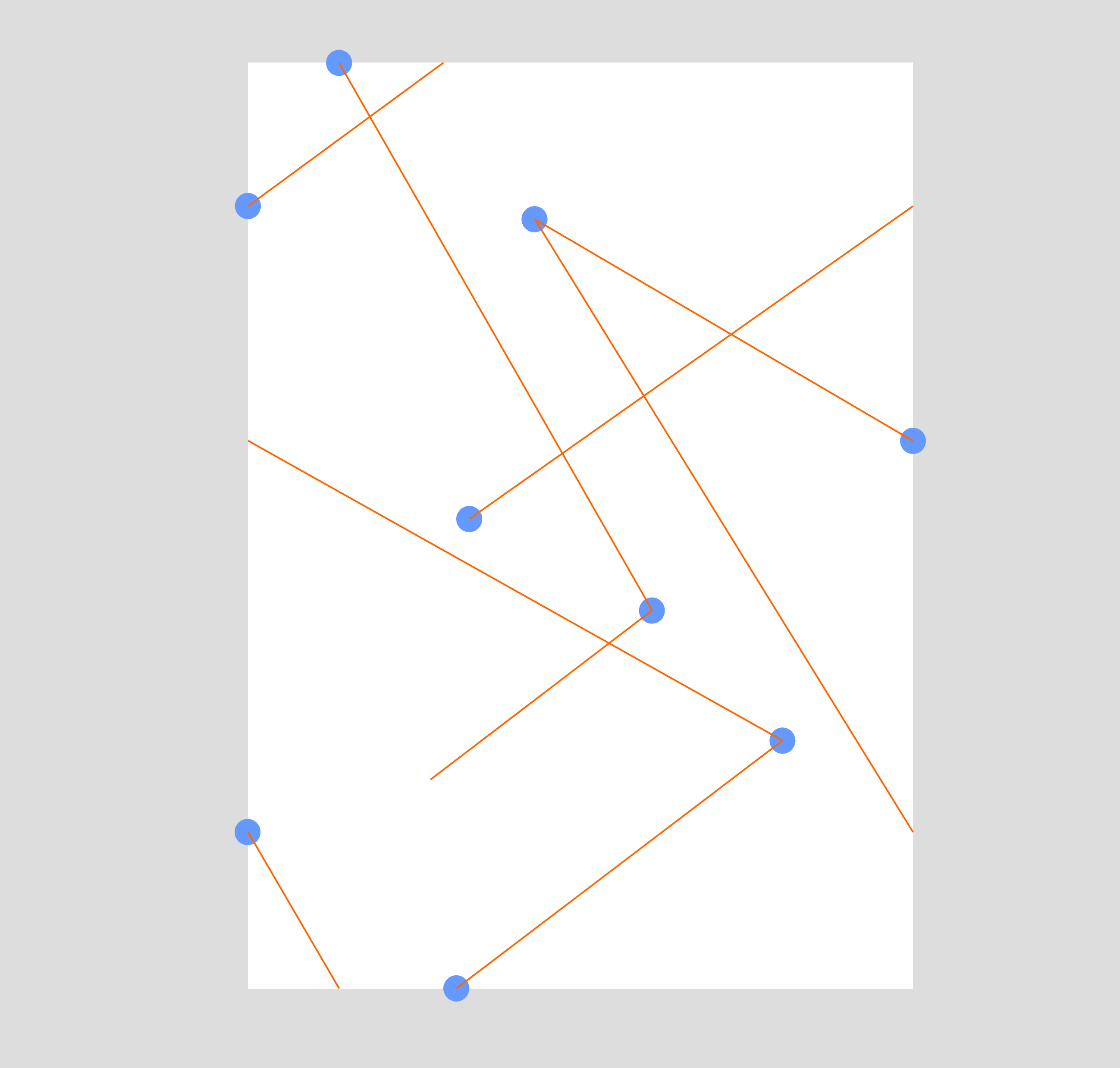
Road maps available from road-map databases

- Ex. US Bureau's TIGER database
  - Houston section
  - Used by PalChaudhuri et al [PLV05]



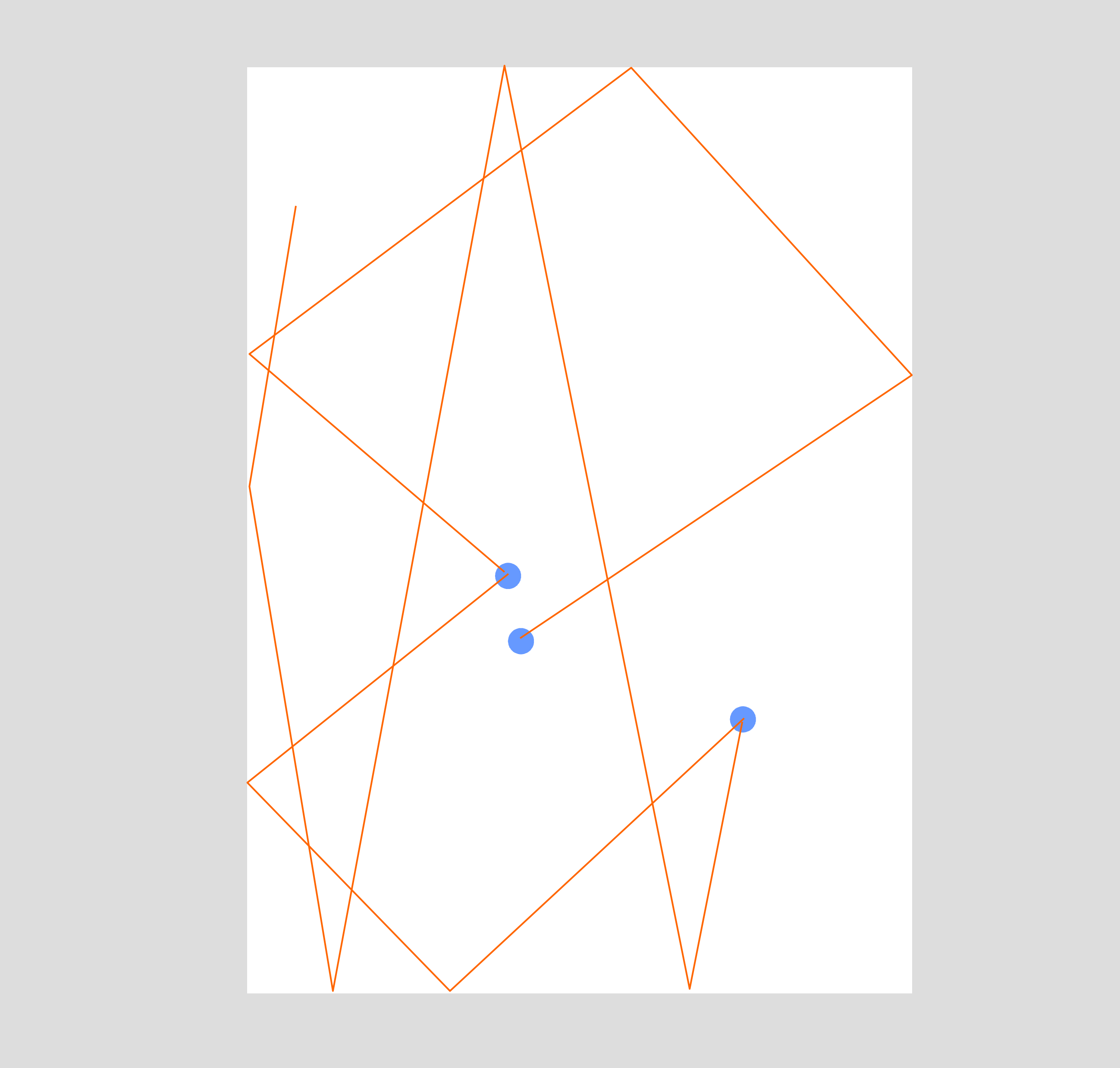
Random walk on torus

- [LV05]
- a.k.a. random direction with wrap around (Nain et al [NT+05])



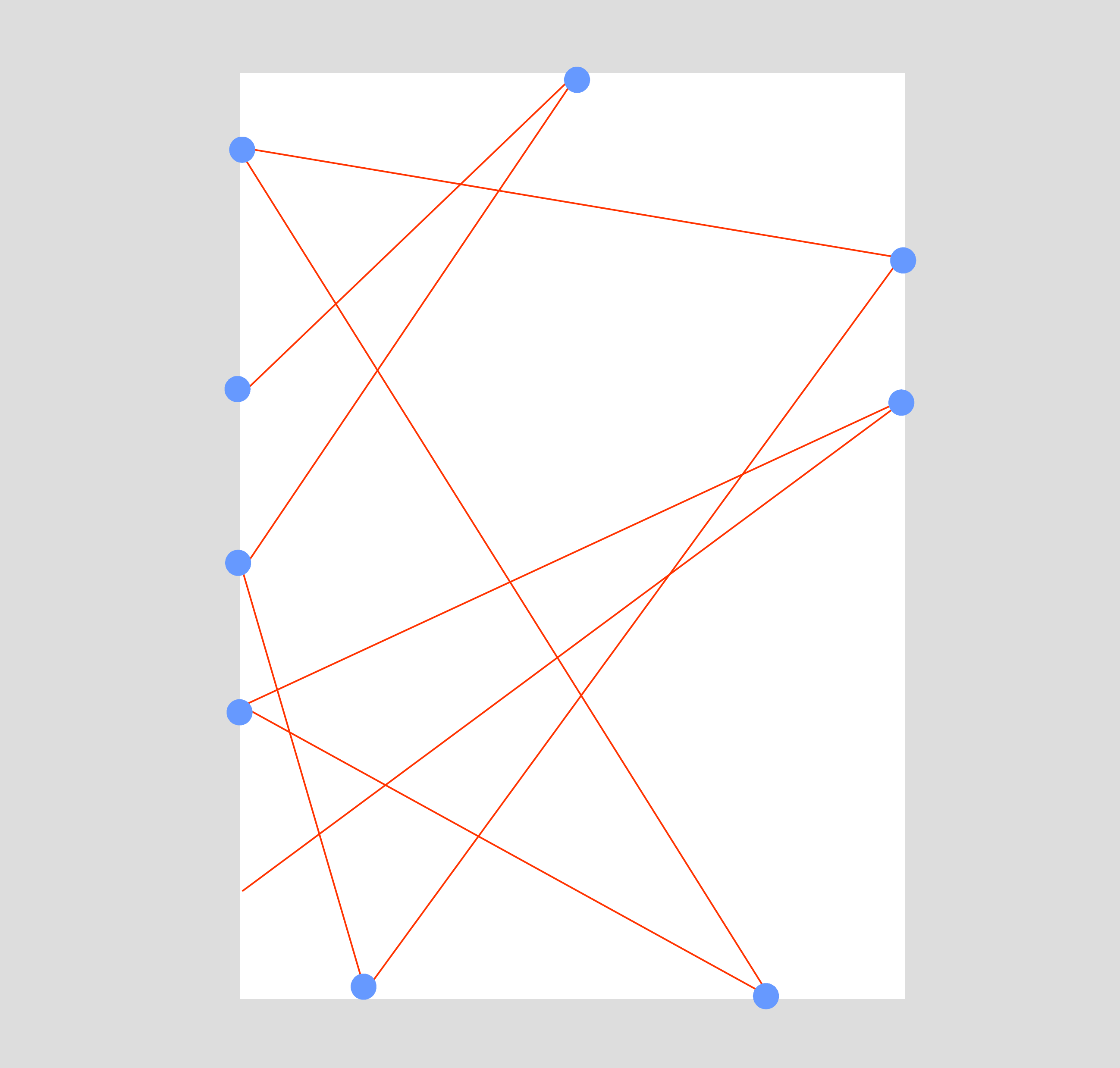
Billiards

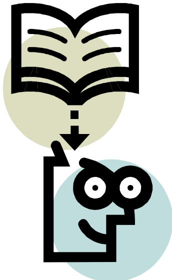
- [LV05]
- a.k.a. random direction with reflection (Nain et al [NT+05])



Stochastic billiards

- Random direction model, Royer et al [RMM01]
- See also survey [CBD02]





Random trip basic constructs » Summary «

- Trip is defined by phase, path, and duration
- The abstraction accommodates many examples
  - Random waypoint on general connected domains
  - Random walk with wrap around
  - Billiards
  - Stochastic billiards

Outline

- Simulation Issues with mobility models
- Random trip basic constructs

- A technical condition: Positive Harris recurrence
- Stability of random trip model
- Time-stationary distributions
- Perfect simulation
- FAQ

An additional condition

- We introduce an additional condition that is needed for stability of random trip to be well understood
  - Positive Harris recurrence
- We check the condition for our catalogue of models 

The Additional condition

- $Y_n = (I_n, P_n)$ (phase, path) is a Markov chain by construction of the random trip model
  - In general, on general state space !
  - Not necessarily bounded or countable
- We assume that Y_n is positive Harris recurrent

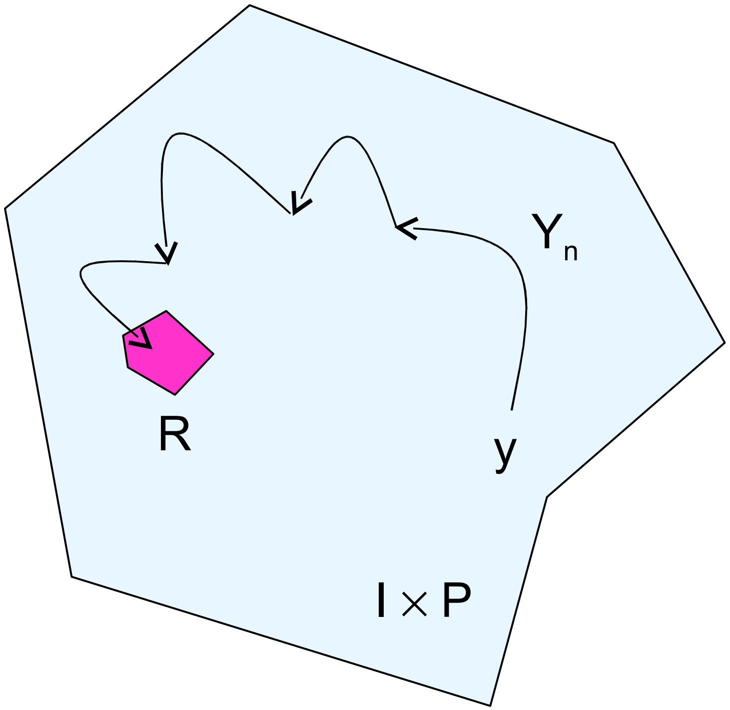
Positive Harris recurrence

- If the state space for the Markov chain of phases and paths would be countable (not true in general), this would mean
  - Any state can be reached
  - No escape to infinity

- A natural condition if we want the mobility state to have a stationary regime

- On a general state space, the definition is more evolved

Harris recurrence

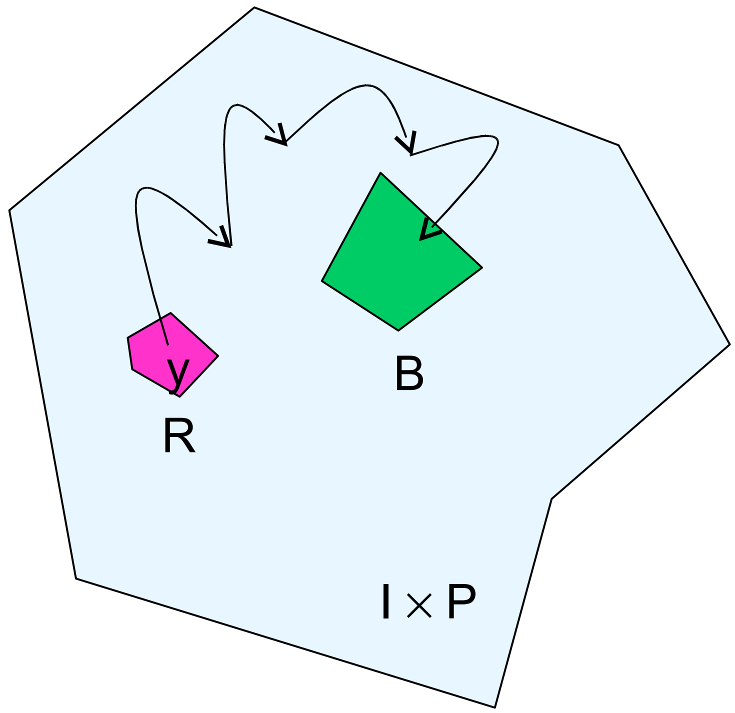


- It means that there exists a set R that is visited by Y_n from any initial state in some given number of transitions

- The set R is "recurrent"

plus …

Harris recurrence (2)



- Probability that Y_n hits a set B starting from R in some given number of transitions is lower bounded by $\beta\ \varphi(B)$

  β is a number in $(0,1)$, φ is a probability measure on $I \times P$

- The set R is “regenerative”

Positive Harris recurrence

- Y_n Harris recurrent implies that Y_n has a stationary measure π^0 on $I \times P$
  - It may be $\pi^0(I \times P) = +\infty$
- We need $\pi^0(I \times P) < +\infty$ so that Y_n has a stationary probability distribution
- We assume that Y_n is positive Harris recurrent
  - It means Harris recurrent plus that the return time to set R has a finite expectation

Check the condition for random waypoint

- For this model, it is easy

- It suffices to consider RWP with no pauses

- Note that any two paths P_n, P_m such that $|n - m| > 1$ are independent

- Hence

$$P(P_n \in A_1 \times A_2 \mid P_0 = p) = |A_1| \cdot |A_2|, \text{ for all } n > 1$$

- Take as the recurrent set $R \equiv A \times A$

Check condition for restricted random waypoint

The condition is true if

- In addition to assumptions for random waypoint, it holds
  - The Markov walk on sub-domains is irreducible
  - And the mean number of trips within a sub-domain is finite

- Proof follows from well known stability results for Markov chains on finite state spaces

Check condition for random walk on torus

The condition is true if

- The speed vector has a density in R^2
- And, trip duration has a density, conditional on either phase is move or pause

Check condition for random walk on torus(2)

- Main thing to prove is that node position at trip transitions, X_n, is Harris recurrent

- Fact: the distribution of X_n started from any given initial point, converges to uniform distribution, provided *only* that node speed *has a density*

- Harris recurrence follows by the latter fact, Erdos-Turan-Koksma inequality, and Fourier analysis

Check condition for billiards 

The condition is true if

- The speed vector has a density in R^2 that is *completely symmetric*
- And, trip duration has a density, conditional on either phase is move or pause

- Proof by reduction to random walk (see [LV06])

- Def. A random vector (X,Y) is said to have a completely symmetric distribution iff (-X,Y) and (X,-Y) have the same distribution as (X,Y)

To be complete …

- We also need to assume:

  (a) Trip duration S_n is strictly positive

  (b) Distribution of trip duration S_n is non-arithmetic

  arithmetic = on a lattice

- These are minor conditions, can in practice be assumed to hold
  - (a) is common sense
  - (b) is true in particular if S_n has a density

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- Simulation Issues with mobility models
- Random trip basic constructs
- A technical condition: Positive Harris recurrence

- Stability of random trip model
- Time-stationary distributions
- Perfect simulation
- FAQ

Stability of random trip model
» Outline «

- What do we mean by stability ?
- We give the stability result for random trip

Stability

- Informally, the model is stable if the distribution of system state converges to something well defined, as the simulation time grows

- If so:
  - "The simulation reaches a stationary regime"
  - There is a well defined time stationary distribution of system state that can be used for fair comparisons

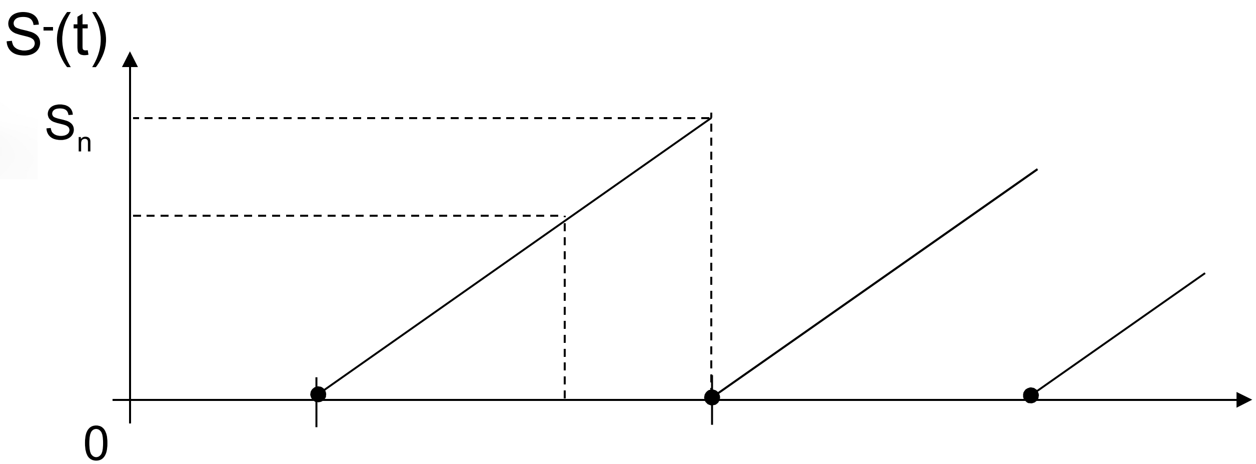
Stability (formal definition)

- System state $\Phi(t) = (Y(t), S(t), S^{-}(t)), \quad t \geq 0$

(phase, path)

duration of current trip

time elapsed on current trip



$\forall$ $\Phi(t)$ has

- A unique time-stationary distribution π
- The distribution of $\Phi(t)$ converges to π as t goes to infinity

Stability of random trip model

- There exists a time-stationary distribution π for $\Phi(t)$ *if and only if mean trip duration is finite* (trip sampled at trip endpoints)
- Whenever π exists, it is unique

Stability of random trip model (2)

- Moreover, if mean trip duration is finite, from any initial state, the distribution of $\Phi(t)$ converges to π as t goes to infinity
- Otherwise, from any initial state the distribution of $\Phi(t)$ converges to 0 as t goes to infinity

Application to random waypoint

- Mean trip duration for a move
  = (mean trip distance) $\times$ mean of inverse of speed

- Mean trip duration for a pause
  = mean pause time

- Random waypoint is stable if both
  - mean of inverse of speed
  - mean pause time

  are finite

A Random waypoint model that has no time-stationary distribution !

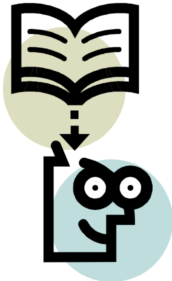
- Assume that at trip transitions, node speed is sampled uniformly on $[v_{min}, v_{max}]$

- Take $v_{min} = 0$ and $v_{max} > 0$ (common in practise)

- Mean trip duration = (mean trip distance) $\times \frac{1}{v_{\max}} \int_0^{v_{\max}} \frac{dv}{v} = +\infty$

- Mean trip duration is infinite !

- Speed decay: “considered harmful” [YLN03]



Stability of random trip model » Summary «

- Random trip model is stable if mean trip duration is finite
- This ensures the model is stable
  - Unique time-stationary distribution, and
  - Convergence to this distribution from any initial state
- Didn't hold for a random waypoint used by many

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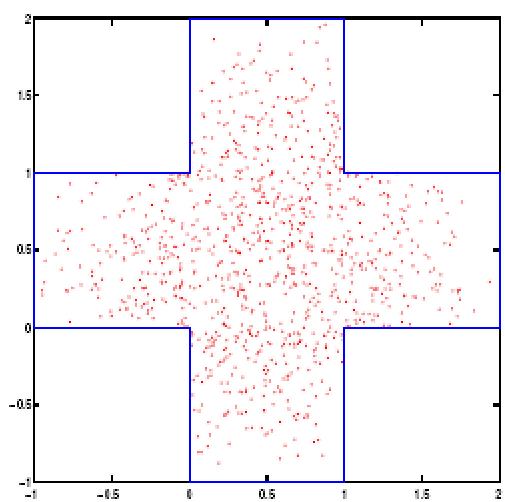
Time-stationary distributions » Outline «

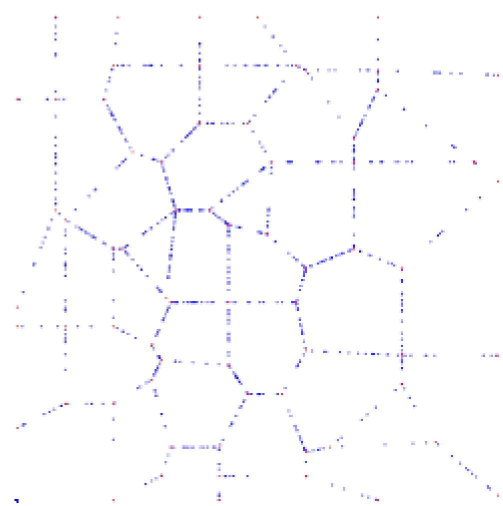
- Time-stationary distribution of node mobility state is the distribution of state in stationary regime
- Should be used for fair comparison
- Can be obtained systematically by the Palm inversion formula
  - Palm inversion formula relates event-stationary distribution at trip transition to time-stationary at arbitrary time

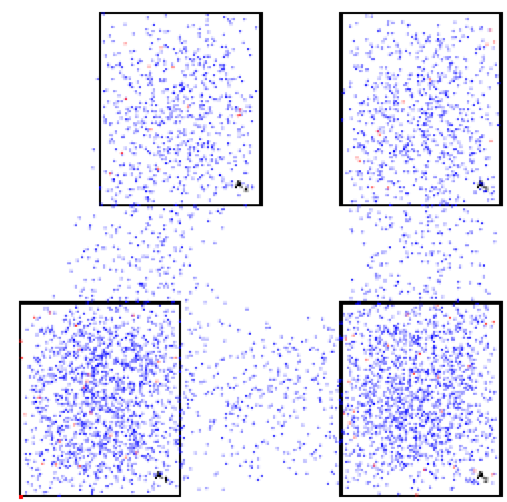
Sampling bias

- Stationary distributions at arbitrary times and at trip end points are not necessarily the same
  - Time-average vs event-average

- Ex. samples of node position for random waypoint
  - Trip endpoints are uniformly distributed, time stationary distribution of mobile location is *not*







Time-stationary distribution given by Palm inversion

- Relates time-averages to event-averages:

$$E(f(\Phi(t))) = \lambda E^0\left[\int_0^{T_1} f(\Phi(s))ds\right] \qquad 1/\lambda = E^0[S_0]$$

Time-average Event-average

- T_n = a trip transition instant
- Time 0 is an arbitrarily fixed time
- Convention: $\ldots T_{-1} \leq T_0 \leq 0 < T_1 \leq \ldots$

Example: random waypoint

- Consider random waypoint with no pauses
- By Palm inversion, we obtain the time-average speed is:

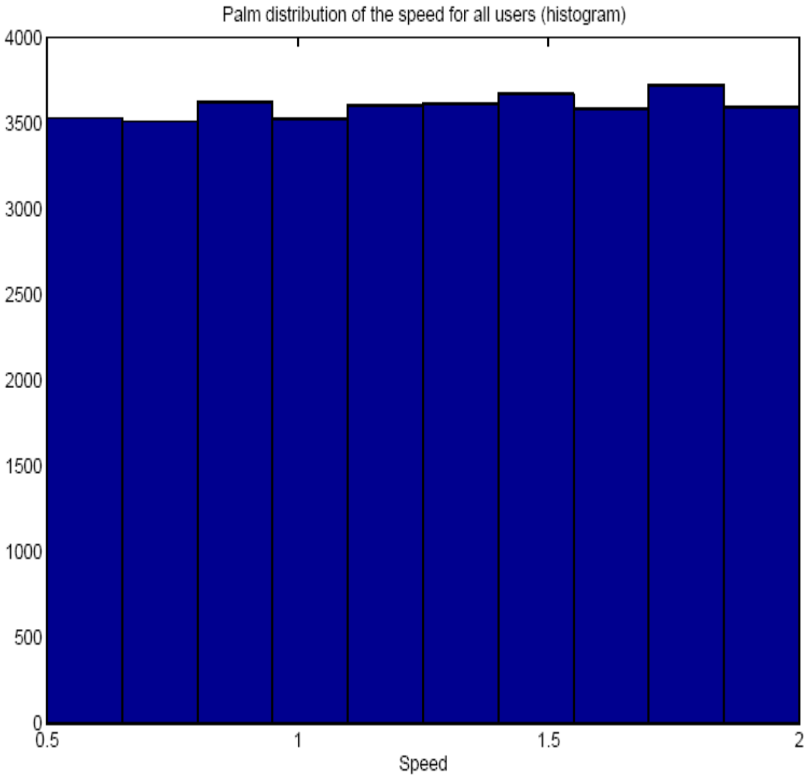
$$E(V(t)) = \frac{E^0\left[\| X_1 - X_0 \|\right]}{E^0\left[\frac{\| X_1 - X_0 \|}{V_0}\right]} = E^0\left[\frac{1}{V_0}\right]^{-1}$$

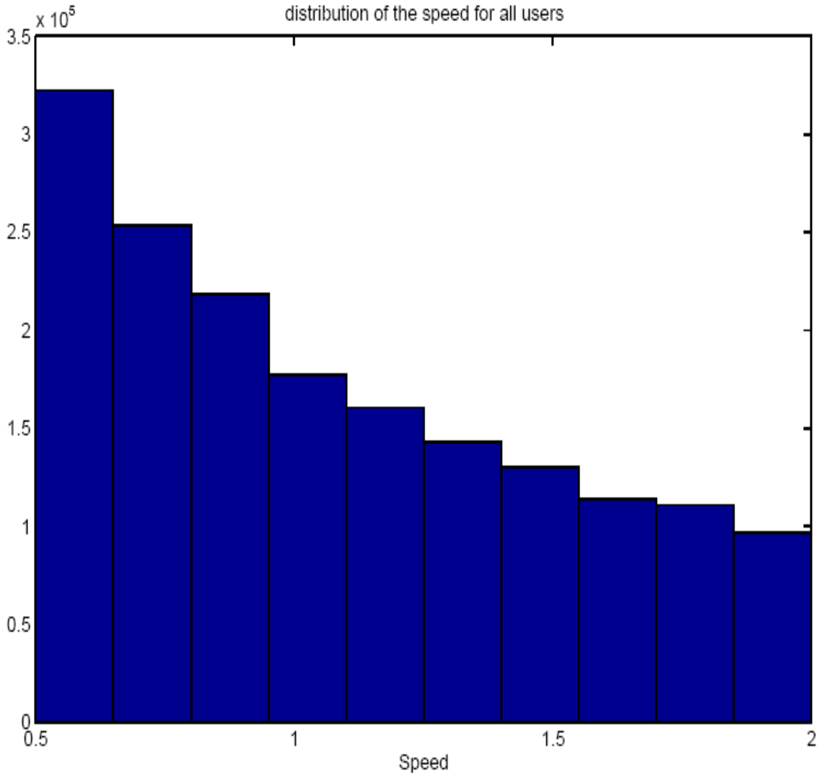
- It follows:

$$\underbrace{f_V(v)}_{\text{Time-stationary speed density}} = \text{const}\,\frac{1}{v}\,\underbrace{f_0^V(v)}_{\text{Event-stationary speed density}}$$

Example: random waypoint (2)

- Histogram of node speed sampled at trip transitions
- Histogram of node speed sampled at equidistant times





Representation of time-stationary distribution (any random trip model)

- Phase:

$$P(I(t) = i) = \frac{\pi^0(i)\tau_i}{\sum_j \pi^0(j)\tau_j}$$

where $\tau_i = E^0(S_0 \mid I_0 = i)$, i.e. mean trip duration given that phase is i

- Path and duration, given the phase:

$$dP(P(t) = p, S(t) = s \mid I(t) = i) = \frac{s}{\tau_i} dP^0(P_0 = p, S_0 = s \mid I_0 = i)$$

- Time elapsed on the current trip: $S^-(t) = S(t)U(t)$, where U(t) is uniform on [0,1]

Time-stationary distribution for (restricted) random waypoint

- Conditional on phase is (i, j, r, move)

- Node speed at time t is independent of path and location with density

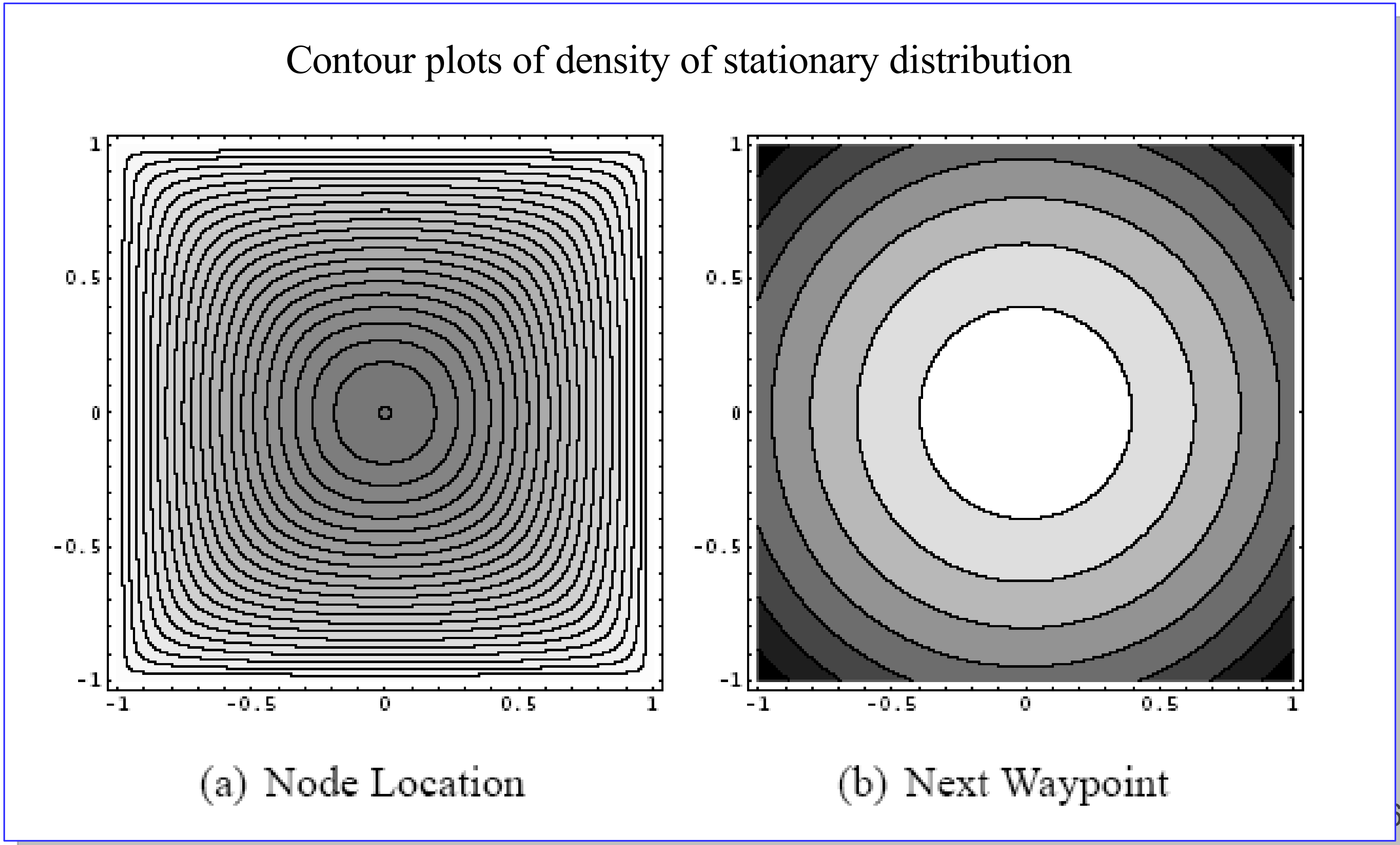
$$f_V(v) = \text{const}\, \frac{1}{v} f_0^V(v)$$

- Path endpoints at time t, (P(t)(0),P(t)(1)) = (m_0,m_1) have a joint density:

$$= K_{ijr}\, d(m_0, m_1), \quad \text{for } (\mathrm{m}_0, \mathrm{m}_1) \in \mathrm{A}_\mathrm{i} \times A_j$$

- Conditional on (P(t)(0),P(t)(1))=(x,y), distribution of node position X(t) is uniform on the segment [x,y]

The stationary distribution of random waypoint can be obtained in closed form [L04]



Closed forms

$$\left\{\begin{array}{l} f_{M(t)}(x,y) = f_{M(t)}(|x|,|y|) \\ \text{if } |x| < |y| \text{ then } f_{M(t)}(x,y) = f_{M(t)}(|y|,|x|) \\ \text{if } 0 \le y \le x \text{ then } f_{M(t)}(x,y) = \frac{15}{32\left(\sqrt{2}+2+5\ln(1+\sqrt{2})\right)} F(x,y) \end{array}\right.$$

with $F(x,y) =$

$$\begin{array}{rrlcrl} & (1-x)(2+x)(1-y) & \sqrt{1+\frac{(1-y)^2}{(1+x)^2}} & + & (1-x)(1-y)(2+y) & \sqrt{1+\frac{(1-x)^2}{(1+y)^2}} \\ + & (1-x)(2+x)(1+y) & \sqrt{1+\frac{(1+y)^2}{(1+x)^2}} & + & (1-x)(1+y)(2-y) & \sqrt{1+\frac{(1-x)^2}{(1-y)^2}} \\ - & \frac{(1-x)^2(1-y)^2}{1+x} & \sqrt{1+\frac{(1+x)^2}{(1-y)^2}} & - & \frac{(1-x)^2(1-y)^2}{1+y} & \sqrt{1+\frac{(1+y)^2}{(1-x)^2}} \\ - & \frac{(1-x)^2(1+y)^2}{1+x} & \sqrt{1+\frac{(1+x)^2}{(1+y)^2}} & - & \frac{(1-x)^2(1+y)^2}{1-y} & \sqrt{1+\frac{(1-y)^2}{(1-x)^2}} \\ + & (1-x)\left[1+x-(1-y)^2\right] & \sinh^{-1}\left(\frac{1-y}{1+x}\right) & + & (1-y)\left[1+y-(1-x)^2\right] & \sinh^{-1}\left(\frac{1-x}{1+y}\right) \\ + & (1-x)\left[1+x-(1+y)^2\right] & \sinh^{-1}\left(\frac{1+y}{1+x}\right) & + & (1+y)\left[1-y-(1-x)^2\right] & \sinh^{-1}\left(\frac{1-x}{1-y}\right) \\ + & (1-x)^2(1-y) & \sinh^{-1}\left(\frac{1+x}{1-y}\right) & + & (1-x)(1-y)^2 & \sinh^{-1}\left(\frac{1+y}{1-x}\right) \\ + & (1-x)^2(1+y) & \sinh^{-1}\left(\frac{1+x}{1+y}\right) & + & (1-x)(1+y)^2 & \sinh^{-1}\left(\frac{1-y}{1-x}\right) \end{array} \quad (14)$$

where $\sinh^{-1}(t) = \ln\left(t+\sqrt{1+t^2}\right)$ *(inverse hyperbolic sine).*

Time-stationary distribution for (restricted) random waypoint (2)

- Conditional on phase is (i, j, r, pause)

- Node location X(t) and residual time until end of pause R(t) are independent

$$f_V(v) = \text{const}\,\frac{1}{v} f_0^V(v)$$

- X(t) is uniform on A_i

- R(t) has density

$$= \frac{1}{\tau_I}\left(1 - F^0_{S|I}(s)\right)$$

Pause time distribution at trip transitions

Time-stationary distribution for random walk on torus

- Node mobility state at time t

  = (I(t), X(t), V(t), R(t))

  I(t) = phase, either move or pause

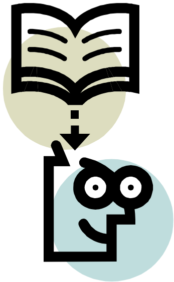
  X(t) = node position

  V(t) = speed vector (= null vector, if I(t) = pause)

  R(t) = residual time until end of trip

Time-stationary distribution for random walk on torus (2)

- Node location X(t) is uniformly distributed
- $P(I(t) = \text{pause}) = \tau_{pause} / (\tau_{pause} + \tau_{move})$
- Conditional on I(t) = pause:
  - R(t) density = $(1-F^0_{pause}(s)) / \tau_{pause}$
  - X(t) and R(t) are independent
- Conditional on I(t) = move:
  - V(t) has density $f^0_V(v)$
  - R(t) density = $(1-F^0_{move}(s)) / \tau_{move}$
  - X(t), V(t), R(t) are independent



Time-stationary distributions » Summary «

- Palm inversion yields systematic characterization of time-stationary distribution for any random trip model

- Closed-form expressions for time-stationary distributions may involve complex geometric integrals
  - But we don’t need them to sample from the time-stationary distributions (see next)

Outline

- Simulation Issues with mobility models
- Random trip basic constructs
- A technical condition: Positive Harris recurrence
- Stability of random trip model
- Time-stationary distributions

- Perfect simulation
- FAQ

Perfect simulation » Outline «

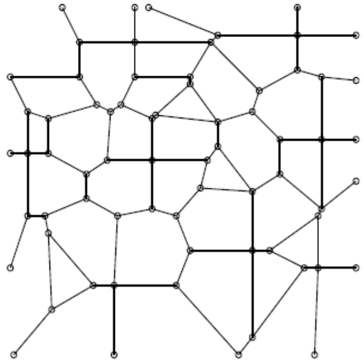
- Perfect simulation
  - Sample initial state from time-stationary distribution
  - Then state is a time-stationary realization at any time

- Perfect sampling algorithm
  - Uses characterization seen earlier
  - Plus rejection sampling
  - No need to compute geometric constants

Perfect simulation is highly desirable

- If model is stable and initial state is drawn from distribution other than time-stationary distribution
  - The distribution of node state converges to the time-stationary distribution
- Naïve: so, let’s simply truncate an initial simulation duration
- The problem is that initial transience can last very long

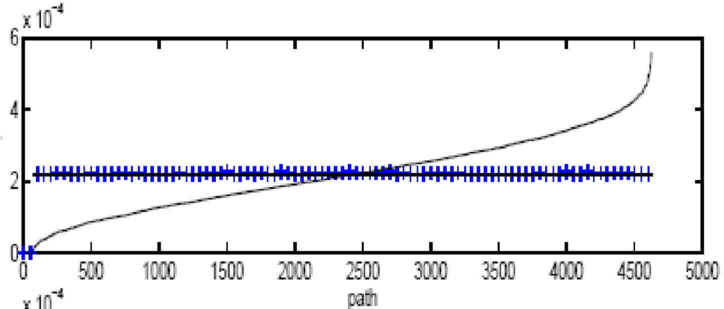
Example [space graph]:
node speed = 1.25 m/s
bounding area = 1km x 1km



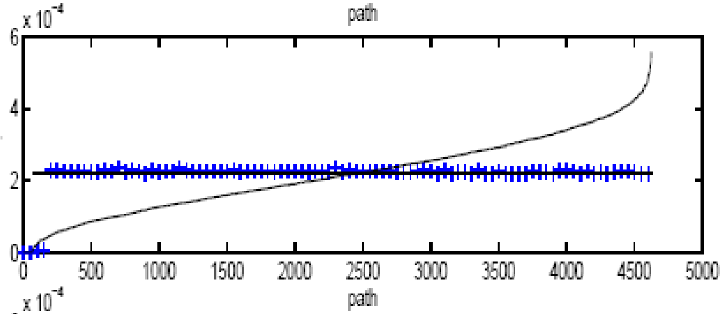
Perfect simulation is highly desirable (2)

- Distribution of path:

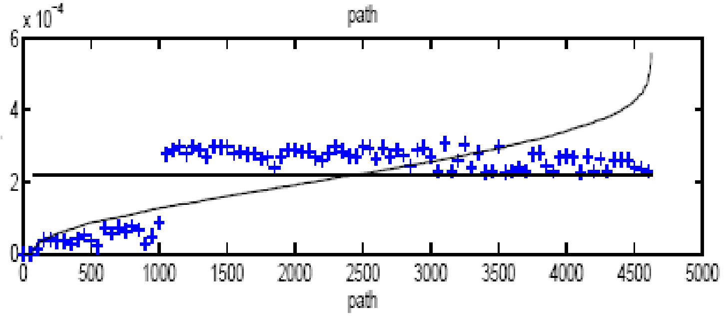
Time = 50s



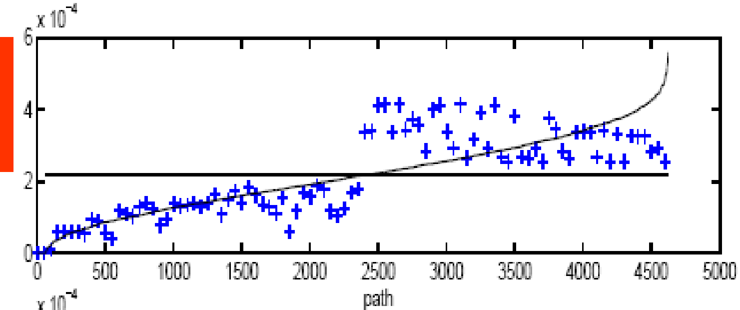
Time = 100s



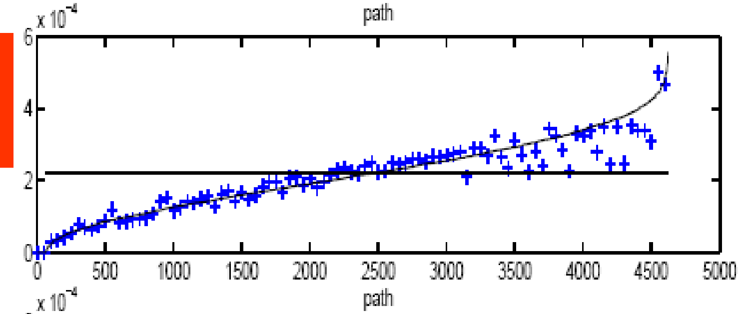
Time = 300s



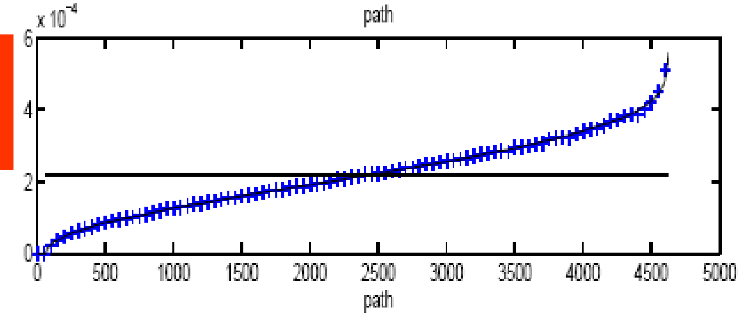
Time = 500s



Time = 1000s



Time = 2000s



Perfect sampling algorithm for random waypoint

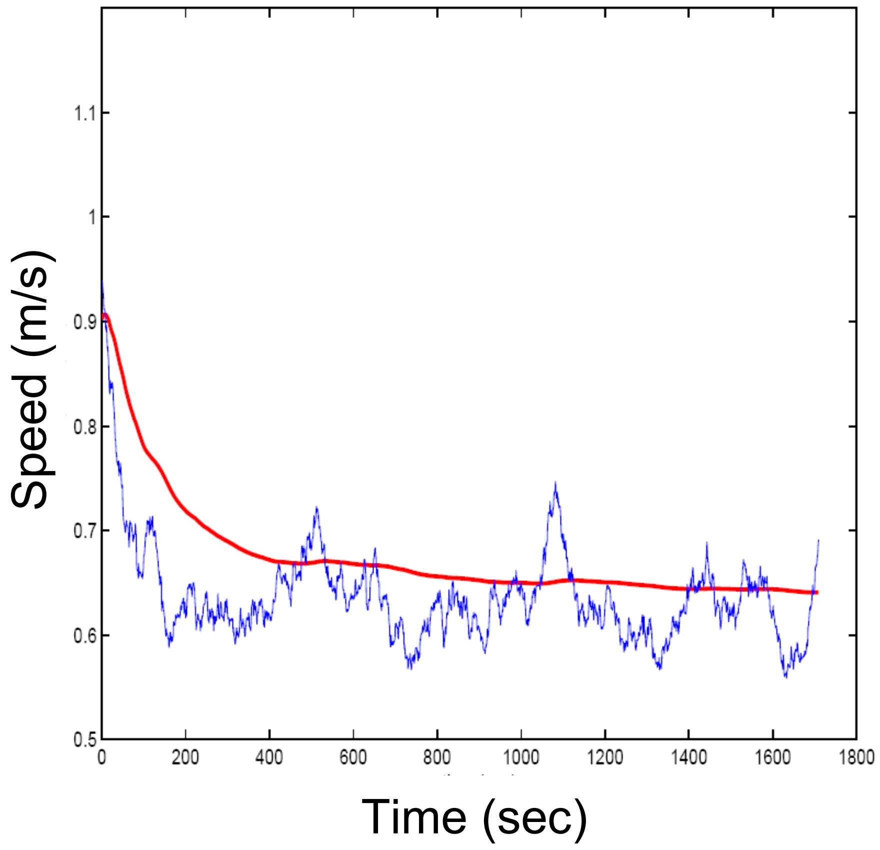
Input: A, Δ
Output: X_0, X, X_1

3. Do
   sample X_0, X_1, iid, $\sim$ Unif(A)
   sample $V \sim$ Unif$[0, \Delta]$
   until $V < ||X_1 - X_0||$
4. Draw $U \sim$ Unif$[0,1]$
5. $X = (1-U)\, X_0 + U\, X_1$

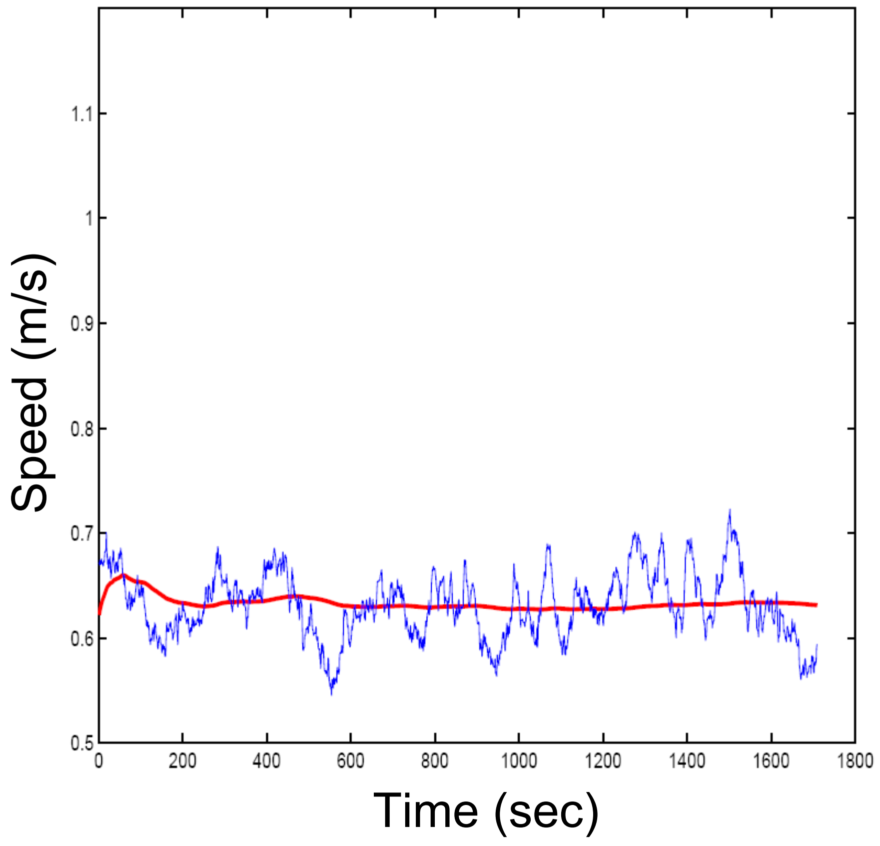
Input: A = domain, Δ = upper bound on the diameter of A

Example: random waypoint
No speed decay

- Standard simulation

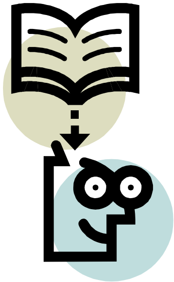


- Perfect simulation



Perfect simulation software

- Developed by Santashil PalChaudhuri
  - see the random trip web page
- Scripts to use as front-end to ns-2
  - Output is ns-2 compatible format to use as input to ns-2
- Supported models:
  - Random waypoint on general connected domain
  - Restricted random waypoint
  - Random walk with wrapping
  - Billiards



Perfect simulation » Summary «

- Random trip model can be perfectly simulated
  - Node mobility state is a time-stationary realization throughout a simulation
- Perfect simulation by rejection sampling
  - It alleviates knowing geometric constants
  - Bound on the trip length is sufficient

Outline

- Simulation Issues with mobility models
- Random trip basic constructs
- A technical condition: Positive Harris recurrence
- Stability of random trip model
- Time-stationary distributions
- Perfect simulation

FAQ

Frequently Asked Questions

- Does model accommodate power-law inter-contact times ?
- Does model accommodate heavy-tailed trip durations ?
- Can model produce a given time-stationary distribution of node position ?
- What are mobility data resources ?

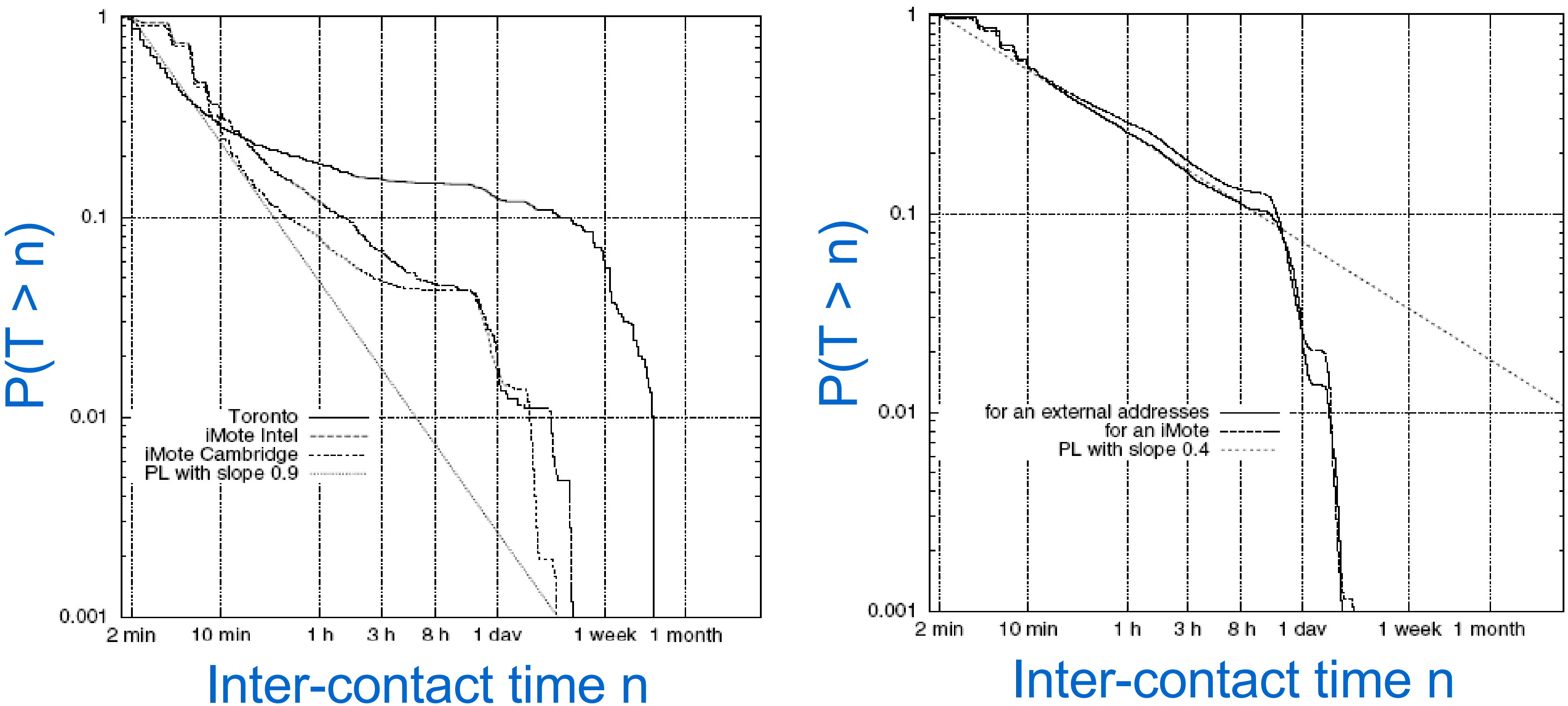
Frequently Asked Questions

Does model accommodate power-law inter-contact times ?

- Does model accommodate heavy-tailed trip durations ?
- Can model produce a given time-stationary distribution of node position ?
- What are mobility data resources ?

Power-law evidence

- Chaintreau et al 2006 [CHC+06]: distribution of inter-contact times of human carried devices (iMote/PDA) is well approximated by a power law



- Source [CHC+06] with permission from authors

Power-law inter-contact times (cont'd)

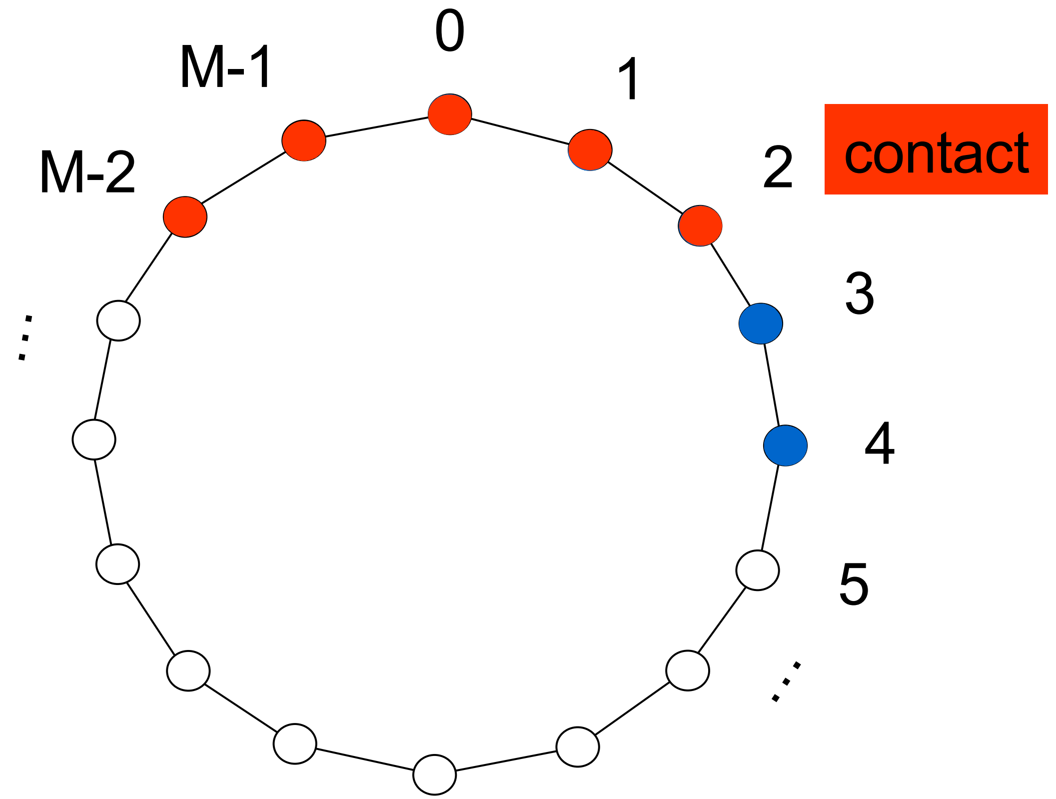
- Implications on packet-forwarding delay ([CHC+06])

?	Can random trip model accommodate power-law node inter-contacts ?

  - Yes ! (see next example)

Example: random walk on torus

- Discrete-time, discrete-space of M sites



- T = inter-contact time, $E(T) = M$

Example: random walk on torus (2)

- Let first $M \to \infty$ (infinite lattice)

  $P(T > n) \sim const / n^{1/2}$, large n

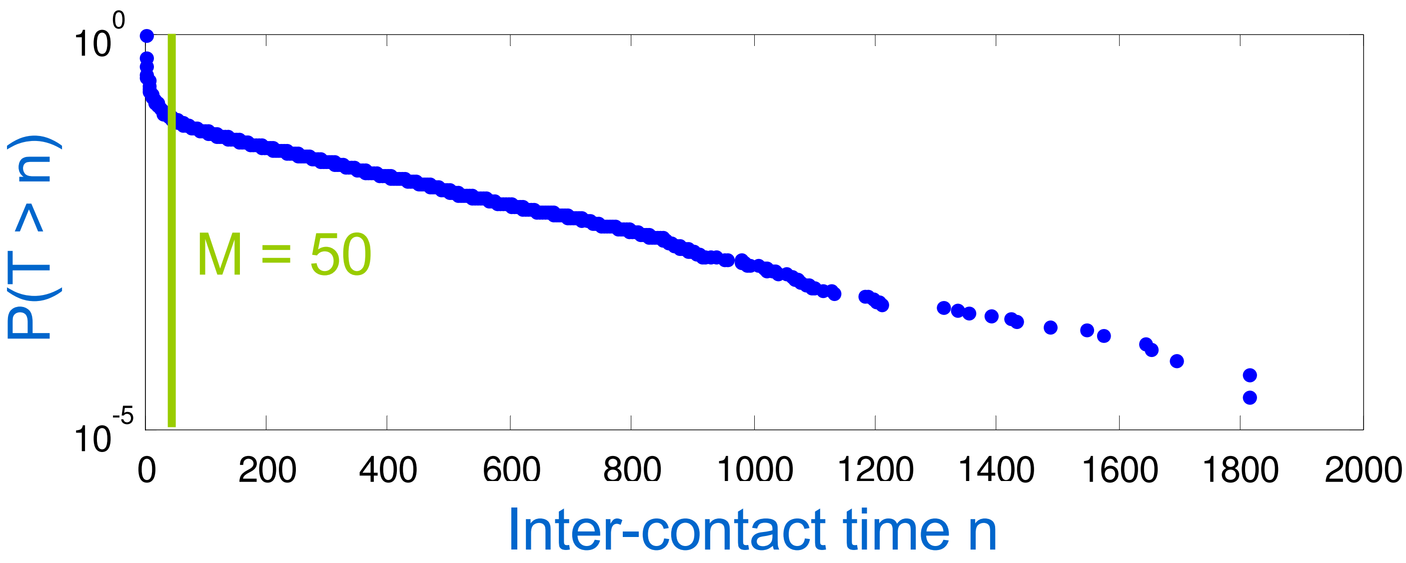
  power-law

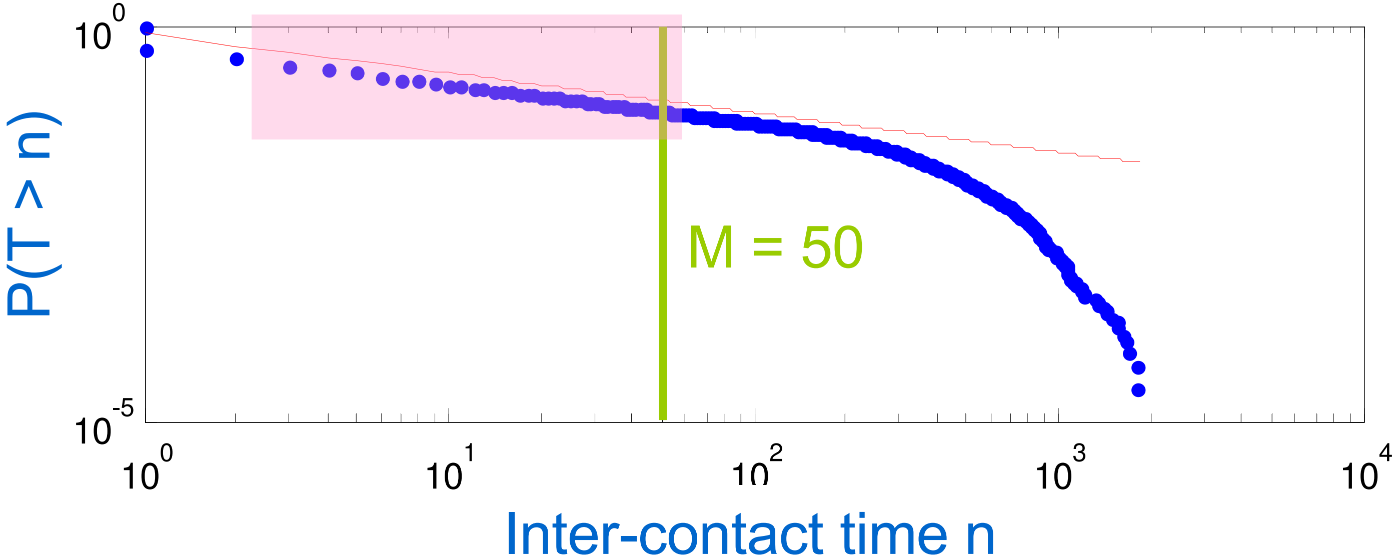
  - Holds for any aperiodic recurrent random walk with finite variance on infinite 1dim lattice, Spitzer [S64]

- If M is fixed, tail is exponentially bounded

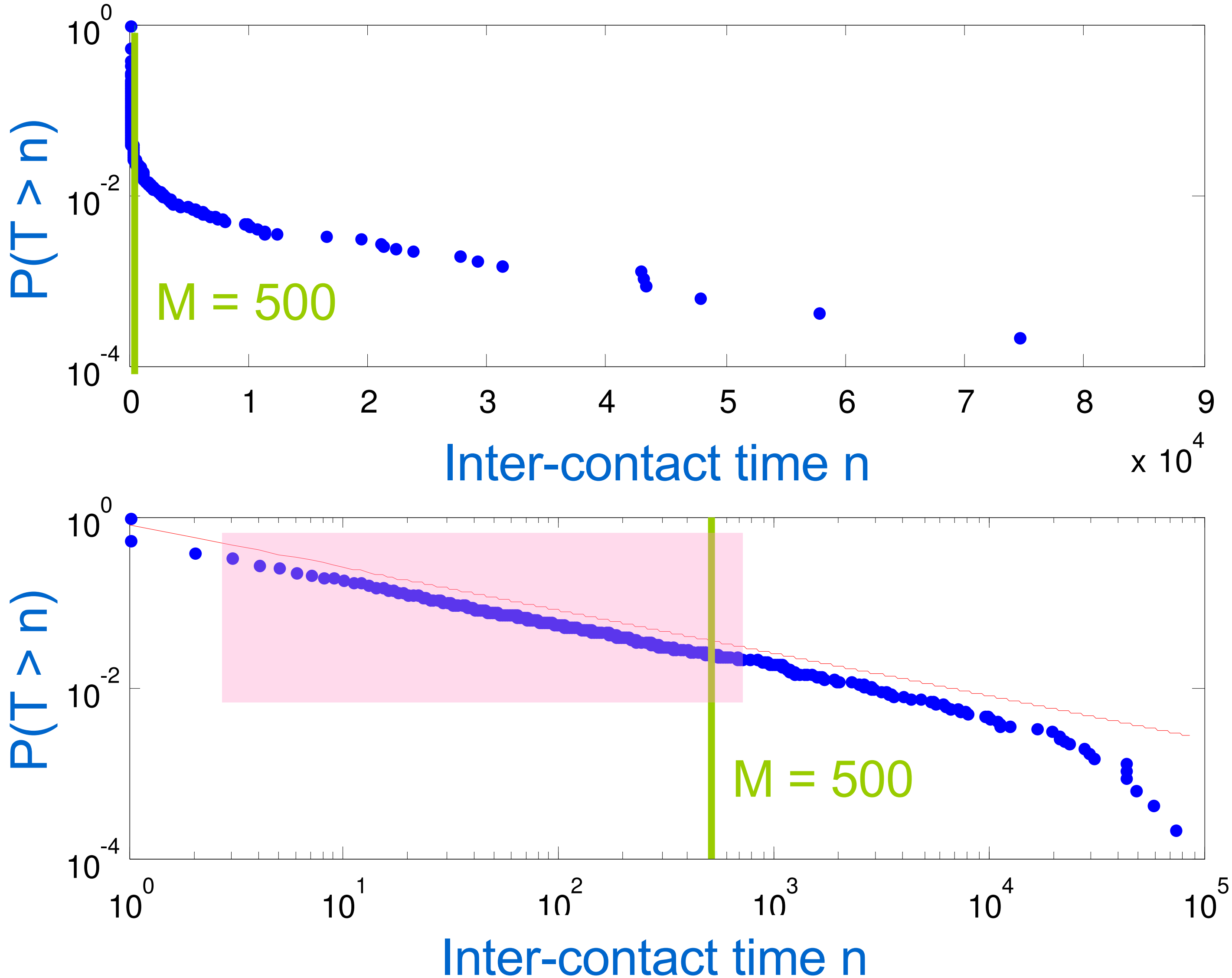
- If n and M scale simultaneously ? (see next)

Example: random walk on torus (3)
M = 50

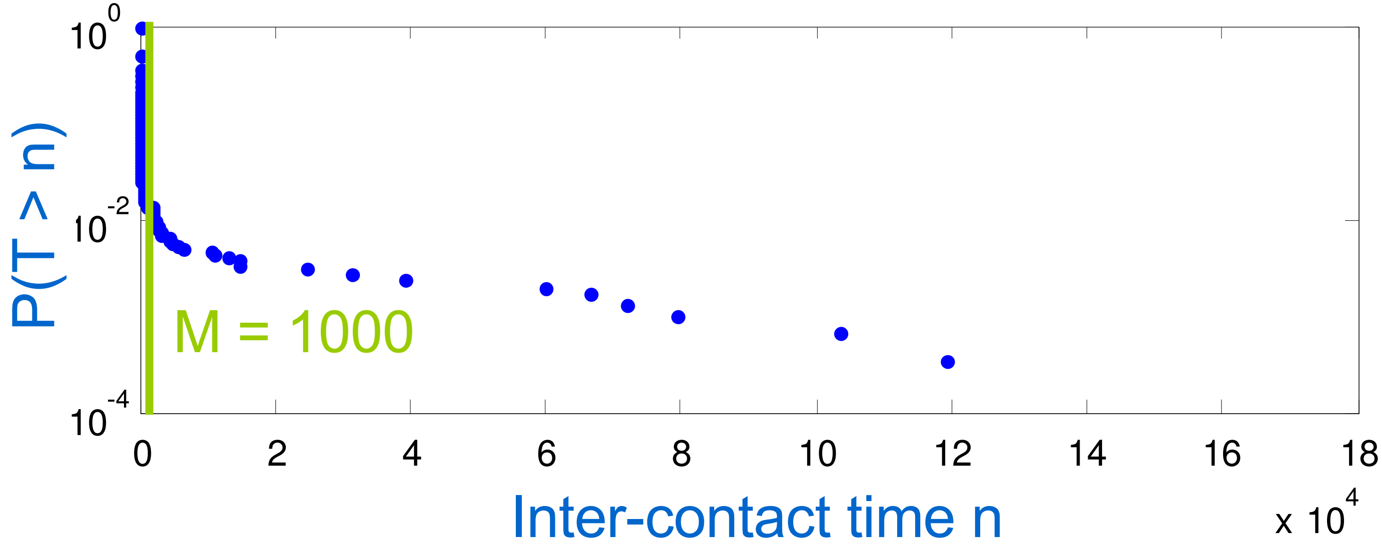


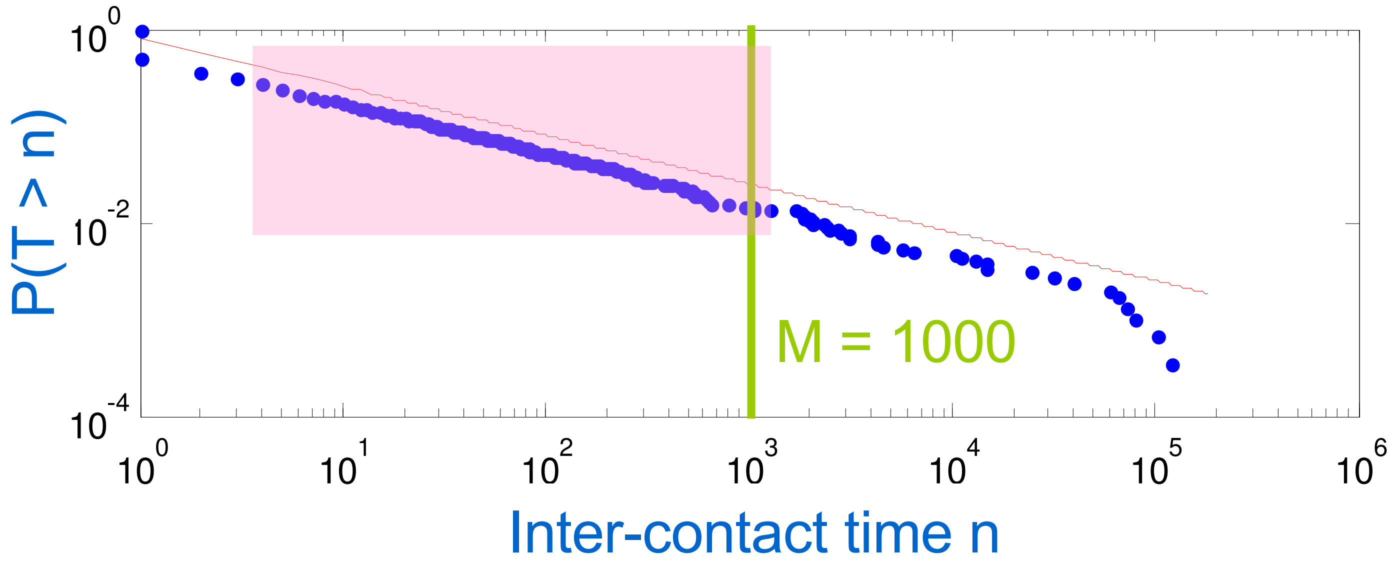


Example: random walk on torus (4) M = 500



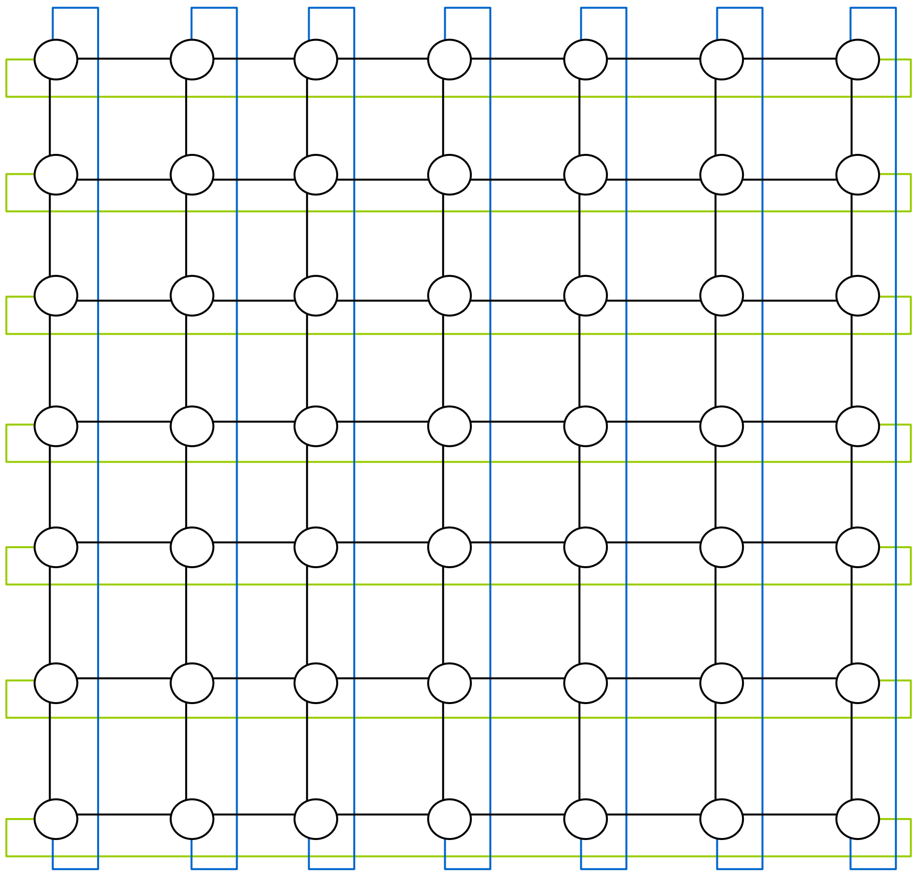
Example: random walk on torus (4) M = 1000





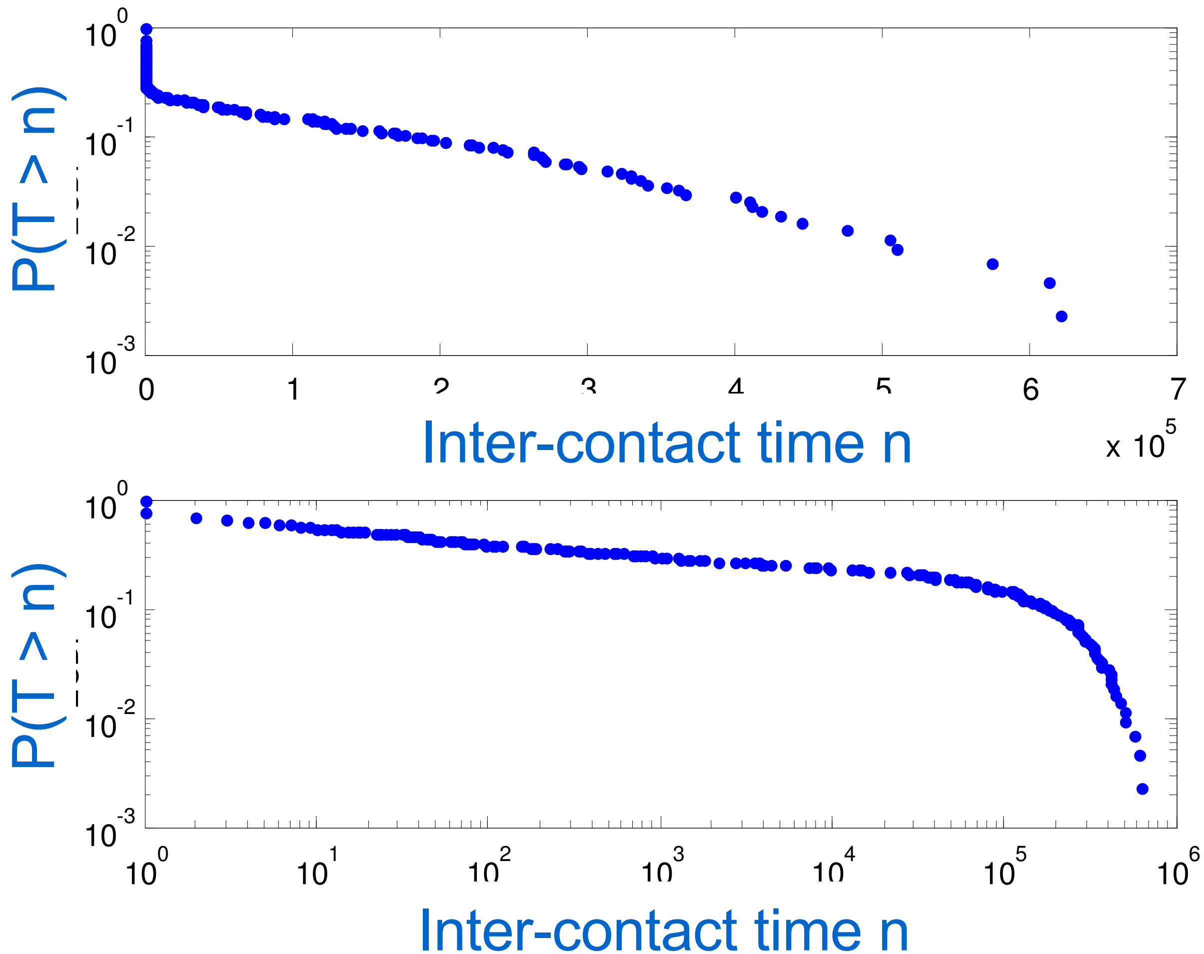
What if random walk is on a 2dim torus ?

- Manhattan grid
- Ex [M87], [SMS06]



What if random walk is on a 2dim torus ? (2)

- Finite torus: 500 x 500 (20M walk steps)



Frequently Asked Questions

- Does model accommodate power-law inter-contact times ?

Does model accommodate heavy-tailed trip durations ?

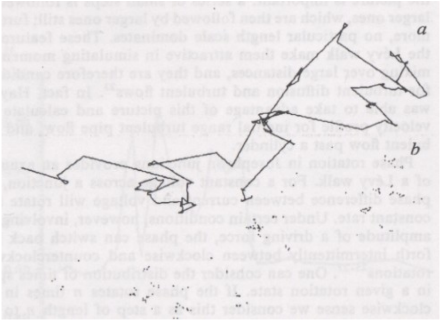
- Can model produce a given time-stationary distribution of node position ?

- What are mobility data resources ?

Heavy-tailed trip times

? Can trip duration be heavy-tailed ?
– Yes.

- Common in nature
  - Albatross search, spider monkeys [KS05], jackals [ARMA02]
  - Model: random walk with heavy-tailed trip distance (Levy flights)



Levy flight (source [FZK93])

Heavy-tailed trip times (2)

- Ex 1: random walk on torus or billiards
  - Take a heavy-tailed distribution for trip duration with finite mean
  - Ex. Pareto: $P^0(S_n > s) = (b/s)^a$, $b > 0$, $1 < a < 2$
- Ex 2: Random waypoint
  - Take $f_V^0(v) = K \; v^{1/2} \; 1(0 \leq v \leq vmax)$
  - $E^0(S_n) < \infty$, $E^0(S_n^2) = \infty$

Frequently Asked Questions

- Does model accommodate power-law inter-contact times ?
- Does model accommodate heavy-tailed trip durations ?

Can model produce a given time-stationary distribution of node position ?

- What are mobility data resources ?

Given time-stationary distribution of node position

- Given is a random trip model with time-stationary density of node position $a_X(x)$

? Can one configure the model so that time-stationary density of node position is a given $b_X(x)$?

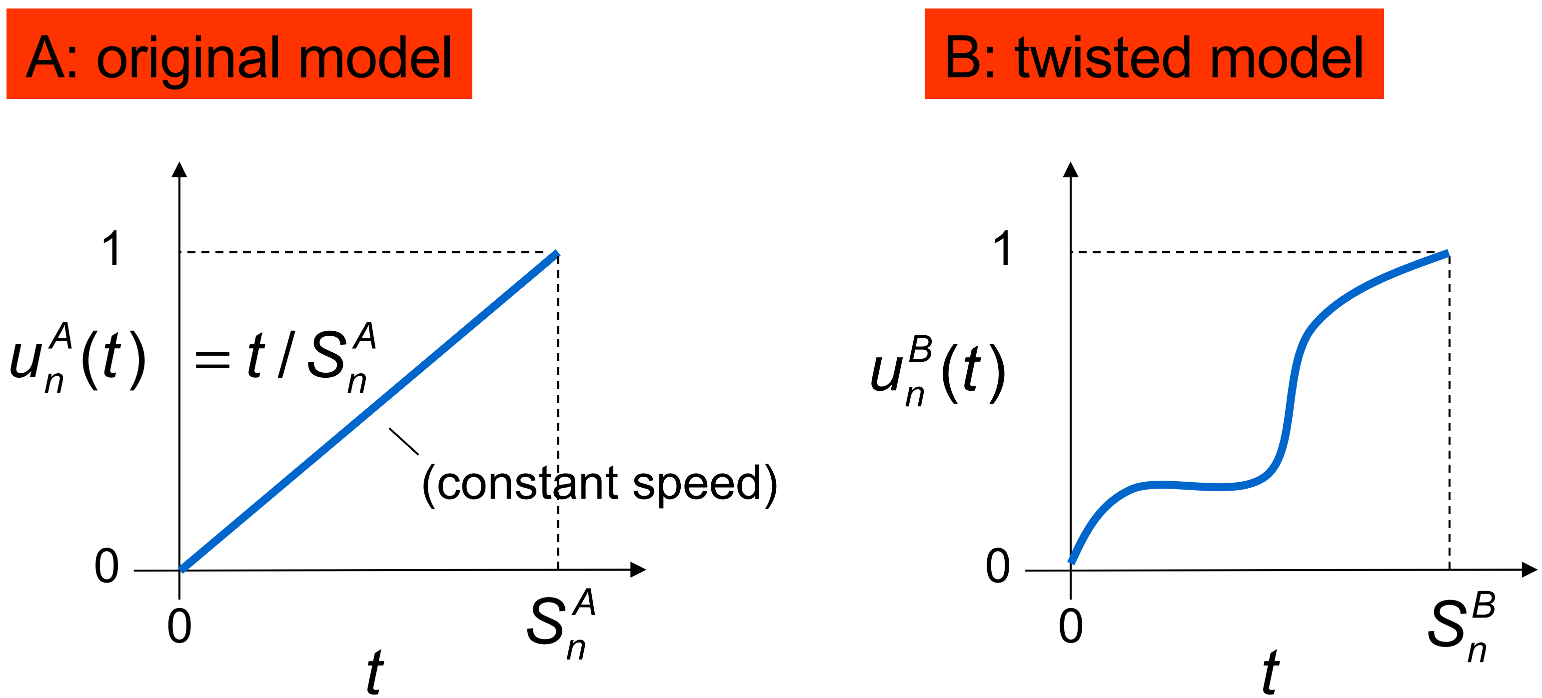
  – Yes. Twist speed as described next

  Remarks:

  – Speed twisting applies to random trip model, in general

  – See [GL06], for random direction model

Speed twist



t = time elapsed on trip

u_n^A, u_n^B = fraction of traversed trip length

- Twist function $u_n^B(t)$?

Speed twist (2)

- Palm inversion formula: the twist function is given by differential equation:

$$\frac{d}{dt} u_n^B = \frac{1}{S_n^A} w\left(P_n\left(u_n^B\right)\right), \quad 0 \le t \le S_n^B$$

with boundary values $u_n(0) = 0$, $u_n(S_n{}^B) = 1$

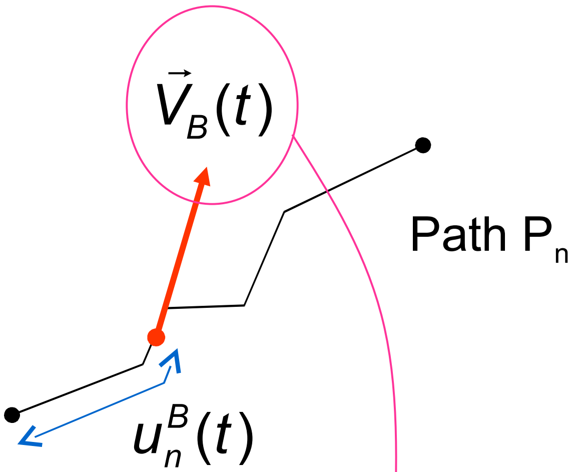
and $w(x) := a_X(x) \,/\, b_X(x)$

- Trip duration may change but its mean remains same:

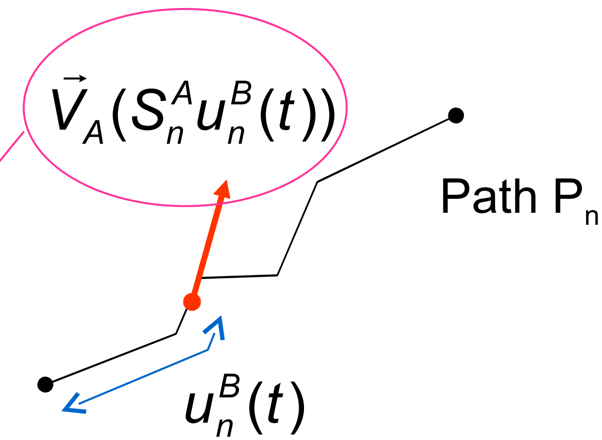
$$E^0(S_0^B) = E^0(S_0^A)$$

Speed twist (3)

B: twisted model



A: original model



$$\vec{V}_B(t) = w(P_n(u_n^B(t))) \times \vec{V}_A(S_n^A u_n^B(t))$$

node location at time t

- At location x, speed is inversely proportional to the target density $b_X(x)$ of location x

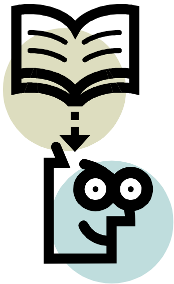
Frequently Asked Questions

- Does model accommodate power-law inter-contact times ?
- Does model accommodate heavy-tailed trip durations ?
- Can model produce a given time-stationary distribution of node position ?

What are mobility data resources ?

Resources

- Partial list:
  - CRAWDAD (crawdad.cs.dartmouth.edu)
  - Haggle (www.haggleproject.org)
  - MobiLib (nile.usc.edu/MobiLib)
  - Street maps:
    - U.S. Census Bureau TIGER database (www2.census.gov/geo/tiger)
    - Mapinfo (www.mapinfo.com)

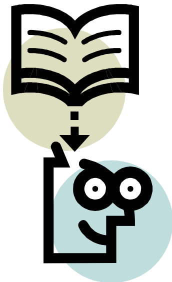


Frequently Asked Questions » Summary «

- Power-law inter-contact times are captured by some random trip models

- Trip duration can be heavy tailed

- Given time-stationary distribution of node position can be achieved



Concluding remarks

- Random trip model covers a broad set of models of *independent* node movements
  - All presented in the catalogue of this slide deck
- Defined by a set of stability conditions
- Time-stationary distributions specified by Palm inversion
- Sampling algorithm for perfect simulation
  - No initial transience
  - Not necessary to know geometric constants

Future work

- Realistic mobility models ?
- Real-life invariants of node mobility ?
  - Human-carried devices, vehicles, …
- What extent of modelling detail is enough ?
- Scalable simulations ?
- Algorithmic implications ?
- Scalable simulations ?
- Statistically dependent node movements
  - Application scenarios, models ?

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