CS3331 Concurrent Computing Exam 2 Solutions
Fall 2020

1. **Synchronization**

(a) **[15 points]** Consider the following solution to the mutual exclusion problem for two processes P_0 and P_1. A process can be making a request `REQUESTING`, executing in the critical section `IN_CS`, or having nothing to do with the critical section `OUT_CS`. This status information, which is represented by an `int`, is saved in `flag[i]` of process P_i. Moreover, variable `turn` is initialized elsewhere to be `0` or `1`. Note that `flag[]` and `turn` are global variables shared by both P_0 and P_1.

```
   int   flag[2];   // global flags, initialized to OUT_CS
   int   turn;      // global turn variable, initialized to 0 or 1

   Process i (i = 0 or 1)

   // Enter Protocol
1. repeat                                      // repeat the following
2.    flag[i] = REQUESTING;                    // making a request to enter
3.    while (turn != i && flag[j] != OUT_CS)   // as long as it is not my turn and
4.       ;                                     //    the other is not out, wait
5.    flag[i] = IN_CS;                         // OK, I am in (well, maybe); but,
6. until flag[j] != IN_CS;                     //    must wait until the other is not in
7. turn = i;                                   // set the turn to mine!

   // critical section

   // Exit Protocol
8. turn = j;                                   // yield the CS to the other
9. flag[i] = OUT_CS;                           // I am out of the CS
```

Prove rigorously using the **proof-by-contradiction** technique that this solution satisfies the mutual exclusion condition. *You will receive* ***zero*** *point if* **(1)** *you prove by example, or* **(2)** *your proof is vague and/or not convincing.*

Answer: A process that enters its critical section must first set `flag[i] = IN_CS` (line 5) and then see `flag[j] != IN_CS` being true at the end of the `repeat-until` loop (line 6). Therefore, we have the following:

- **Condition for P_0 to enter its critical section**: If process P_0 is in the critical section, it had executed `flag[0] = IN_CS` followed by seeing `flag[1] != IN_CS`. That is, `flag[0] = IN_CS` **and** `flag[1] != IN_CS` are true.
- **Condition for P_1 to enter its critical section**: If process P_1 is in the critical section, it had executed `flag[1] = IN_CS` followed by seeing `flag[0] != IN_CS`. Hence, `flag[1] = IN_CS` **and** `flag[1] != IN_CS` both hold.
- **Prove by Contradiction**: If P_0 and P_1 are both in their critical sections, then (`flag[0] = IN_CS` **and** `flag[1] != IN_CS`) **AND** (`flag[1] = IN_CS` **and** `flag[0] != IN_CS`) hold at the same time. However, this means that `flag[0]` and `flag[1]` are equal to and not equal to `IN_CS` at the same time. This is absurd. Consequently, mutual exclusion hold.

Note that the variable `turn` does not play a role here. Right after P_0 and P_1 pass their `repeat-until` loop, they will store some value to `turn`. At this point, because P_0 and P_1 will be in their critical sections without any obstruction, and the value in `turn` does not matter.

See page 10 of `06-Sync-Soft-Hardware.pdf`. This is the same technique as the one we used to show that Attempt II satisfies the mutual exclusion condition. ■

(b) **[15 points]** Consider the following solution to the mutual exclusion problem for two processes P_0 and P_1, where `status[ ]` is a Boolean array of two elements and `turn` is an integer variable. Furthermore, there are three constants indicating the status of a process, where `COMPETING`, `IN_CS` and `OUT_CS` mean competing to enter the critical section, in the critical section, and out of the critical section. Note that `status[ ]` and `turn` are global variables shared by both processes.

```
int  status[2];   // status of a process
int  turn;        // initialized to either 0 or 1

         Process 0                     Process 1
         =========                     =========
 1 status[0] = COMPETING;           status[1] = COMPETING;
 2 do {                             do {
 3    while (turn != 0) {              while (turn != 1) {
 4       status[0] = OUT_CS;              status[1] = OUT_CS;
 5       if (status[turn] == OUT_CS)      if (status[turn] == OUT_CS)
 6          turn = 0;                        turn = 1;
 7    }                                }
 8    status[0] = IN_CS;               status[1] = IN_CS;
 9 } while (status[1] == IN_CS);    } while (status[0] == IN_CS);
                      // critical section
10 status[0] = OUT_CS;              status[1] = OUT_CS;
```

Use a clear and convincing execution sequence to show that this solution does not satisfy the **bounded waiting** condition. A convincing argument **is** required. *You will receive* **zero** *point if* **(1)** *you do not use a valid execution sequence, or* **(2)** *your execution sequence is vague and/or unconvincing.* **Hint:** the value of `turn` plays a significant role.

Answer: As indicated in the hint, the role of `turn` is significant. Note that `turn` is only set to 0 and 1 by P_0 and P_1 (line 6), respectively. Upon exit its critical section, from P_0's point of view `turn` is zero. This is because P_0 must break the `while` loop, which means `turn` is 0. Now, if P_0 comes back fast before P_1 can set `turn` to 1, P_0 can enter immediately. Therefore, if P_0 (*resp.*, P_1) exits its critical section, `turn` is 0 (*resp.*, 1). This is the major cause of the failure of the bounded waiting condition. In other words, this solution is in favor of the just-exited process.

	P_0	P_1	`turn`	`status[0]`	`status[1]`	*Comment*
1			0			
2	`s[0]=C`	`s[1]=C`	0	`C`	`C`	Entering
3	`while`	`while`	0	`C`	`C`	P_0 breaks `while`
4	`s[0]=IN`		0	`IN`	`C`	P_0 about to enter
5		`s[1]=OUT`	0	`IN`	`OUT`	P_0 about to enter
6	**P_0 enters its critical section**					
7	`s[0]=OUT`	`if`	0	`OUT`	`OUT`	P_0 enters CS
8	**P_0 comes back**					
9	`s[0]=C`	`while` loops back	0	`C`	`OUT`	P_0 entering
10		`if`	0	`C`	`OUT`	`if` is false
11	`while`	`while`	0	`C`	`OUT`	P_1 loops back
12	`s[0]=IN`		0	`IN`	`OUT`	P_0 about to enter
13	**P_0 enters its critical section**					

Suppose that `turn` is 0, meaning P_0 may just exit its critical section. See the execution sequence above. To save space, we use `s[ ]`, `C`, `IN` and `OUT` to indicate `status[ ]`, `COMPETING`, `IN_CS` and `OUT_CS`, respectively. From the above observation, if P_0 is fast enough so that every time before P_1 can test the value of `status[turn]` P_0 sets `status[0]` to either `COMPETING` or `IN_CS`, P_0 enters. In this execution sequence, P_0 simply repeats the action between line 6 and line 12 in the above execution sequence. As a result, P_1 will starve, which means P_0 simply keeps entering the critical section and P_1 does not have any chance. Of course, the bounded waiting condition fails. ■

(c) **[10 points]*** Define the meaning of a *race condition*? Answer the question first and use execution sequences with a clear and convincing argument to illustrate your answer. **You must explain step-by-step why your example causes a race condition. Without using valid execution sequences you will receive 0 point.**

Answer: A *race condition* is a situation in which *more than one* processes or threads manipulate a shared resource *concurrently*, and the result depends on *the order of execution*.

The following is a simple counter updating example discussed in class. The value of `count` may be 9, 10 or 11, depending on the order of execution of the **machine instructions** of `count++` and `count--`.

```
int          count = 10;  // shared variable

Process 1                 Process 2

count++;                  count--;
```

The following execution sequence shows a race condition. Two processes run concurrently (condition 1). Both processes access the shared variable `count` concurrently (condition 2) because `count` is accessed in an interleaved way. Finally, the computation result depends on the order of execution of the `SAVE` instructions (condition 3). The execution sequence below shows the result being 9; however, switching the two `SAVE` instructions yields 11. Since all conditions are met, we have a race condition. **Note that you have to provide TWO execution sequences, one for each possible result, to justify the existence of a race condition.**

`Thread_1`	`Thread_2`	*Comment*
do somthing	do somthing	`count = 10` initially
`LOAD count`		`Thread_1` executes `count++`
`ADD #1`		
	`LOAD count`	`Thread_2` executes `count--`
	`SUB #1`	
`SAVE count`		`count` is 11 in memory
	`SAVE count`	Now, `count` is 9 in memory

Stating that "`count++` followed by `count--`" or "`count--` followed by `count++`", even using machine instructions, produces different results and hence a race condition is **incomplete**, because the two processes do not access the shared variable `count` concurrently. Note that the use of higher-level language statement interleaved execution may not reveal the key concept of "sharing" as discussed in class. Therefore, use instruction level interleaved instead.

See pp. 5–12 of `05-Sync-Basics.pdf`. ■

2. **Semaphores**

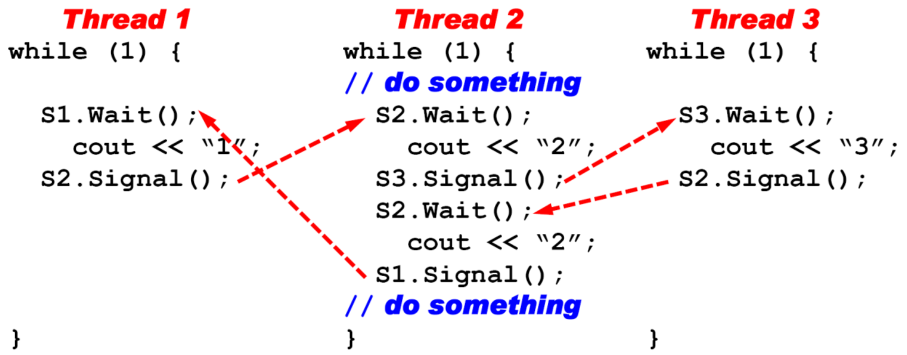
   (a) **[10 points]** Consider the following code:

```
Thread 1               Thread 2               Thread 3
========               ========               ========
while (1) {            while (1) {            while (1)
   // do something        // do something        // do something
   cout << "1";           cout << "2";           count << "3";
   // do something        // do something        // do something
}                      }                      }
```

There are three threads. The first, second, and third thread prints `1`, `2` and `3`, respectively. Declare and add semaphores to the above code so that the output of these three concurrently running threads is `1`, `2`, `3`, `2`, `1`, `2`, `3`, `2`, `1`, ...

Answer: This is a very simple problem. It was discussed in class to some degree and was an exercise on a weekly reading list. The following is a possible solution:

```
Semaphore  S1 = 1, S2 = 0, S3 = 0;
```



```
Thread 1             Thread 2               Thread 3
while (1) {          while (1) {            while (1) {
                     // do something
  S1.Wait();           S2.Wait();             S3.Wait();
    cout << "1";         cout << "2";           cout << "3";
  S2.Signal();         S3.Signal();           S2.Signal();
                       S2.Wait();
                         cout << "2";
                       S1.Signal();
                     // do something
}                    }                      }
```

In the above code, going from thread 1 to thread 2 and going from thread 2 to thread 3 are exactly the same as the "`1 2 1 2 ...`" pattern discussed in class. What is really needed is a going-back pattern from thread 3 to thread 2 and then from thread 2 to thread 1. Note that the `S2.Signal()` call in *Thread 1* will not release the second `S2.Wait()` in *Thread 2.* If Thread 2 is blocked by its second `S2.Wait()`, it has already passed the first `S2.Wait()`, which also means *Thread 2* was released by the `S2.Signal()` call in *Thread 1.* As a result, *Thread 1* is blocked by its `S1.Wait()` and has no way to release *Thread 2.* We need to add a section of code to bridge between thread 3 and thread 1. That is it! ■

(b) **[10 points]** We discussed in class that the two methods `Wait()` and `Signal()` of a semaphore must be atomic to ensure a correct implementation of mutual exclusion. Use execution sequences to show that if `Wait()` is not atomic then *mutual exclusion cannot be maintained.* **You must show clearly what the intended *mutual exclusion* is and how the mutual exclusion condition is violated with execution sequences and provide a convincing explanation. Otherwise, you will risk a lower score. Note also that this question asks for a possible violation of mutual exclusion rather than having a race condition.**

Answer: If `Wait()` is not atomic, its execution may be switched in the middle. If this happens, mutual exclusion will not be maintained. Consider the following solution to the critical section problem:

```
Semaphore S = 1;

Process A              Process B
---------              ---------

Wait(S);               Wait(S);
        // in critical section
Signal(S);             Signal(S);
```

The execution sequence below is a possible example, where `Count = 1` is the internal counter variable of the involved semaphore `S`.

Process A	*Process B*	`Count`	*Comment*
		1	Initial value
`LOAD Count`		1	*A* executes `Count--` of `Wait()`
`SUB #1`		1	
	`LOAD Count`	1	*B* executes `Count--` of `Wait()`
	`SUB #1`	1	
	`SAVE Count`	0	*B* finishes `Count--`
`SAVE Count`		0	*A* finishes `Count--`
`if (Count < 0)`		0	It is false for *A*
	`if (Count < 0)`	0	It is false for *B*
Both *A* and *B* enter the critical section			

Note that this question asks you to demonstrate a violation of mutual exclusion. Consequently, you receive low grade if your demonstration is not a violation of mutual exclusion. Additionally, if you failed to indicate how the needed critical sections that require mutual exclusion is formed, you also risk a lower grade.

This problem was assigned as an exercise in class. See slide 8 of `08-Semaphores.pdf`. ■

(c) **[10 points]** Show that the 1-weirdo solution to the dining philosophers problem will not cause circular waiting and hence is deadlock free. **You should prove this rigorously. A vague and/or unconvincing argument is not acceptable and will receive no points.**

Answer: Suppose the weirdo is philosopher 5. We have two cases to consider: **(1)** if Philosopher 5 has his right chopstick, and **(2)** if Philosopher 5 does not have his right chopstick.

- **Philosopher 5 Has His Right Chopstick:**
  In this case, Philosopher 1 does not have his left chopsticks and cannot eat. See Figure (a) below. The worst case is that philosophers 2, 3 and 4 all have their left chopsticks and are waiting for their right ones in order to eat. Otherwise, there is at least one philosopher is eating, and the system does not have a deadlock. Depending on whether Philosopher 5 has his left chopstick, we have the following two possibilities.

  - **Philosopher 5 Has His Left Chopstick:**
    In this case, Philosopher 5 has both chopsticks and can start eating. There is no deadlock. See Figure (b) below.
  - **Philosopher 5 Does Not Have His Left Chopstick:**
    In this case, Philosopher 5's left chopstick is being held by Philosopher 4 as his right chopstick, and, of course, Philosopher 4 can eat. There is no deadlock either. See Figure (c) below.
- **Philosopher 5 Does Not Have His Right Chopstick:**
  In this case, Philosopher 5's right chopstick is being held by Philosopher 1 as his left chopstick, and, hence, Philosopher 5 cannot eat. See Figure (d) below. The worst case possible is that Philosopher 1 to Philosopher 4 all have their left chopsticks and wait for their right chopsticks. Since Philosopher 5 cannot eat, his left chopstick is free and can be used by Philosopher 4 as his right chopstick. Therefore, Philosopher 4 can eat, and there is no deadlock.

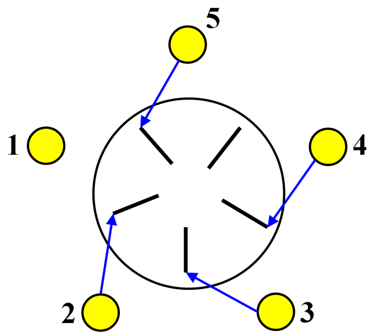


Figure (a)

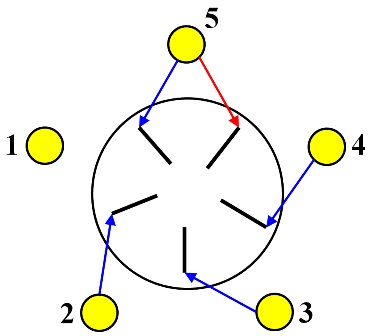


Figure (b)

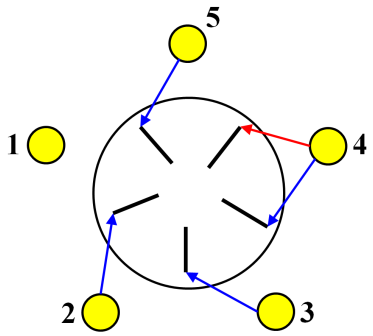


Figure (c)

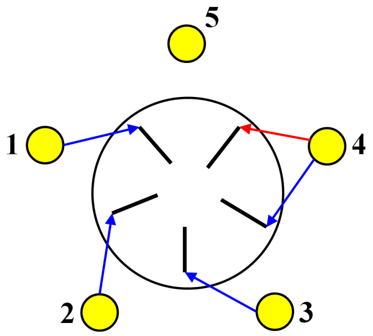


Figure (d)

Since the above enumeration is exhaustive and since none of these cases can cause a deadlock, we conclude that the 1-weirdo version is deadlock free. ■

3. **Problem Solving:**

   (a) **[15 points]** Let T_0, T_1, …, T_{n-1} be n threads, and let `a[ ]` be a global `int` array of n elements. Moreover, thread T_i only uses `a[i]` and `a[(i+1)%n]` for $0 \leq i \leq n-1$. Thus, array `a[ ]` is "circular." Additionally, `Center` is a global variable shared by all threads.

   The code below processes `Center` and `a[ ]` **without** synchronization,

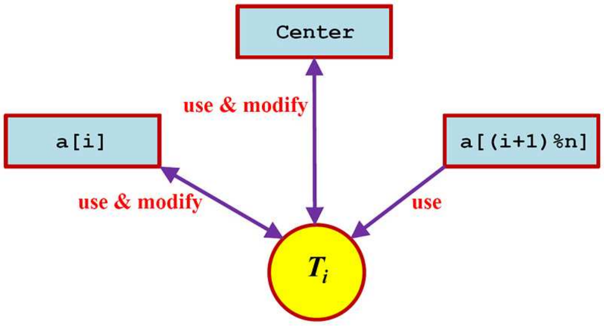
```
int  Center = ...       // some initial value
int  a[n]   = { .... }  // some initial values

Thread i
========

while (1) {
   // other irrelevant computation
   a[i]   = f(a[i], a[(i+1)%n]); // f() does not use a[ ] and Center
   Center = a[i] + Center;
   // other irrelevant computation
}
```

Declare and insert semaphores to the above code so that the indicated task can be performed correctly. You may use as many semaphores as you want. However, you should avoid **busy waiting**, **race conditions**, and **deadlocks**. Moreover, your modification should aim for **maximum concurrency**. A code with close to minimum concurrency receives **zero** point.

Answer: The code template shows three important facts: **(1)** variable `Center` is being shared by all threads, **(2)** array element `a[(i+1)%n]` is being used by thread T_i and $T_{(i+1)\%n}$, and **(3)** array element `a[i]` is **modified** by thread T_i. See the diagram below.



Therefore, array `a[ ]` resembles the chopsticks in the Dining Philosophers problem, and each array element (*i.e.*, chopstick) has to be protected by a semaphore. Thread *i* (*i.e.*, philosopher *i*) uses `a[i]` and `a[(i+1)% n]`. Finally, a semaphore is needed to protect `Center`. Let these semaphores be `S_Center` and `S_a[ ]`. We have the following:

```
int  Center = ...       // some initial value
int  a[n]   = { .... }  // some initial values

Semaphore S_Center = 1;
Semaphore S_a[n]   = { 1, 1, ....., 1 };
```

It is important to note that thread T_i only reads `a[(i+1)% n]`. It is not worth to lock `a[(i+1)% n]` while in function `f( )`, which could be time consuming. As a result, we could lock `a[(i+1)% n]` and copy its value to a local variable `Local`. In this way, `a[(i+1)% n]` is free for thread $T_{(i+1)\%n}$ to use (actually modify). Because only T_i modifies `a[i]` after calling `f( )`, the execution of `f(a[i],Local)` does not affect the value of `a[i]` and no protection for `f( )` is needed. When saving the result of `f( )` to `a[i]`, protection is needed. The final version looks like the following:

```
     int  Center = ...       // some initial value
     int  a[n]   = { .... }  // some initial values

     Semaphore S_Center = 1;
     Semaphore S_a[n]   = { 1, 1, ....., 1 };

     Thread i
     ========
 1.  int Local, fx;

 2.  while (1) {
 3.     // other irrelevant computation
 4.     S_a[(i+1)%n]).Wait();        // Lock a[(i+1)%n]
 5.        Local = a[(i+1)%n];       // Make a copy
 6.     S_a[(i+1)%n]).Signal();      // Release a[(i+1)%n]
 7.     fx = f(a[i], Local);         // f() does not use a[ ] and Center
 8.     S_a[i].Wait();               // Lock a[i]
 9.        a[i] = fx;                // Update a[i]
10.     S_a[i].Signal();             // Unlock a[i]
11.     S_Center.Wait();             // Lock Center
12.        Center = fx + Center;     // Update Center
13.     S_Center.Signal();           // Release Center
14.     // other irrelevant computation
15.  }
```

The following version is not very efficient. Because `Center` is being used by all threads, at any time there is one and only one thread can be executing Line 6-7 below. Consequently, this serializes the execution of all threads, and the execution of these *n* threads becomes sequential. This means that there is no concurrency at all.

```
     int  Center = ...       // some initial value
     int  a[n]   = { .... }  // some initial values

     Semaphore S_Center = 1;
     Semaphore S_a[n]   = { 1, 1, ....., 1 };

     Thread i
     ========
 1.  while (1) {
 2.     // other irrelevant computation
 3.     S_a[(i+1)%n]).Wait();        // Lock a[(i+1)%n]
 4.        S_a[i].Wait();            // Lock a[i]
 5.           S_Center.Wait();       // Lock Center;
 6.              a[i] = f(a[i], a[(i+1)%n]);
 7.              Center = a[i] + Center;
 8.           S_Center.Signal();     // Release Center;
 9.        S_a[i].Signal             // Release a[i]
10.     S_a[(i+1)%n].Signal          // Release a[(i+1)%n]
11.     // other irrelevant computation
12.  }
```

The following version is even worse:

```
       int  Center = ...       // some initial value
       int  a[n]   = { .... }  // some initial values

       Semaphore S = 1;

       Thread i
       ========
    1. while (1) {
    2.    // other irrelevant computation
    3.    S.Wait();
    4.       a[i] = f(a[i], a[(i+1)%n]);
    5.       Center = a[i] + Center;
    6.    S.Signal();
    7.    // other irrelevant computation
    8. }
```

In terms of concurrency, this version is the same as the above one because there is one and only one thread can be executing in Line 4-5. ■

(b) **[15 points]** A unisex bathroom is shared by men and women. A man or a woman may be using the room, waiting to use the room, or doing something else. They work, use the bathroom and come back to work. The rule of using the bathroom is very simple: *there must never be a man and a woman in the room at the same time; however, people with the same gender can use the room at the same time.*

Man Thread

```
void Man(void)
{
   while (1) {
      // working
      // use the bathroom
}
```

Woman Thread

```
void Woman(void)
{
   while (1) {
      // working
      // use the bathroom
}
```

Declare semaphores and other variables with initial values, and add `Wait()` and `Signal()` calls to the threads so that the man threads and woman threads will run properly and meet the requirement. Your implementation should not have any busy waiting, race condition, and deadlock, and should aim for maximum parallelism.

A convincing correctness argument is needed. Otherwise, you will receive no credit for this problem.

Answer: This is a simple variation of the reader-priority readers-writers problem. More precisely, we allow the "writers" to write simultaneously, the "readers" and "writers" cannot do their work at the same time. Therefore, the writers have the same structures as the readers. We need to maintain two counters, one for the males `MaleCounter` and the other for the females `FemaleCounter`. Of course, we need two Mutexes `MaleMutex` and `FemaleMutex` for mutual exclusion. In addition, there is a semaphore `BathRoom` to block the males (*resp.*, females) if the room is being used by the females (*reap.*, males). Note that the male thread and female thread are symmetric.

```
int       MaleCounter = 0, FemaleCounter = 0;   // male and female counters
Semaphore MaleMutex = 1, FemaleMutex = 1;       // male and female counters
Semaphore BathRoom = 1;                         // the bathroom is empty initially

Male Thread                     Female Thread

while (1) {                     while(1) {
   // working                      // working

   MaleMutex.Wait();               FemaleMutex().Wait();        // update counter
      MaleCounter++;                  FemaleCounter--;
      if (MaleCounter == 1)           if (FemaleCounter == 1)   // if I am the first
         BathRoom.Wait();                BathRoom.Wait();       //    yield to other
   MaleMutex.Signal();             FemaleMutex.Signal();

   // use the bathroom             // use the bathroom

   MaleMutex.Wait();               FemaleMutex.Wait();         // update counter
      MaleCounter--;                  FemaleCounter--;
      if (MaleCounter == 0)           if (FemaleCounter == 0)  // if I am the last one
         BathRoom.Signal();              BathRoom.Signal();    //    let the other group know
   MaleMutex.Signal();             FemaleMutex.Signal();
}                               }
```

Refer to the class notes for the solution to the reader-priority version of the readers-writers problem for the details. ∎